From income to wealth inequality in the U.S.

General equilibrium matters

Job Market Paper

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Abstract

The past 40 years have been characterized by a decrease in the rate of return on safe assets, an increase in the equity premium, an increase in the price of financial assets, and an increase in labor income and wealth inequality. Using a heterogeneous-agent model featuring permanent labor income inequality, a two-asset structure, and non-homothetic preferences, we investigate the impact of an increase in permanent labor income inequality on wealth inequality. As rich households save a higher share of their permanent income than poorer ones, a more skewed permanent labor income distribution increases aggregate savings, everything else equal. However, in general equilibrium, with a realistic market structure, an increase in aggregate savings increases mostly the price of capital, not its quantity. This has little impact on the marginal productivity of capital and labor but creates capital gains that push up the top 1% wealth share.

Keywords: Inequality, permanent income, secular stagnation, interest rates, Tobin's Q

JEL codes: E21, E22, E43

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Introduction

Over the past five decades, both labor income and wealth inequality have increased rapidly in the U.S. (Saez & Zucman 2016, Piketty 2015, Piketty & Zucman 2015, Piketty et al. 2018). This paper aims to explore the relationship between the distribution of permanent labor income and the distribution of wealth. A mechanism highlighted by (Straub 2019) suggests that since households with higher permanent labor income have higher saving rates than poorer households, an increase in permanent labor income inequality can result in an increase in aggregate savings. This change in aggregate savings will, in turn, affect the price of financial assets, which will have a feedback effect on the distribution of wealth. The central question we address is: what is the general equilibrium effect of such an increase in aggregate savings on the distribution of wealth? Does the general equilibrium feedback dampen or amplify wealth inequality?

The answer to this question is theoretically ambiguous. On the one hand, an increase in aggregate savings can increase the *quantity* of physical capital, which will increase wages and decrease returns. This general equilibrium effect should have a "trickle-down" effect and dampen wealth inequality, as poor households will benefit from higher wages, while rich households will see their financial income decrease.[1] On the other hand, an increase in aggregate savings can also increase the *price* of capital. This should increase wealth inequality: wages and long-term returns would remain constant, while short-term capital gains would temporarily increase the returns of the wealthy. To assess the strength of those different channels, we build a quantitative heterogeneous agent model that can account for the relationship observed in the data between the saving rate and the level of permanent income on the household side and the valuation effects on the firm side. Using that framework, we find that the general equilibrium effect amplifies wealth inequality and represents 10% of the total increase in the top 1% wealth share.

We first present some stylized facts on the accumulation of wealth in the U.S. Using a decomposition similar to (Piketty & Zucman 2014), we find that most of the increase in the wealth-to-output ratio in the U.S. came from a valuation effect and not a quantity ef-

[1] As shown by (Davila et al. 2012), in a standard Aiyagari-model calibrated to match the wealth and income distribution in the U.S., there is too little capital at the steady state, and a central planner would want to push the richest to save more, to increase the productivity of all workers.

fect. This is consistent with the fact that the aggregate households' net saving rate has decreased over the same period: households have accumulated wealth mostly by benefiting from large unrealized capital gains on their existing wealth, not by saving a larger share of their income.[2] Instead, an increase in the value of the capital stock has taken place: the average Tobin's Q surged from around 0.6 in the 1960s to over 1 in recent times (Brun & Gonzalez 2017, Gonzalez & Trivin 2019). These aggregate macroeconomic trends also align with micro evidence on saving rates along the wealth distribution. Fagereng et al. (2020) found, using Norwegian register data, that the disparity in gross saving rates between the richest and the rest of the distribution is primarily explained by unrealized capital gains.

Building on those findings, we study a simple two-agent analytical model with different permanent labor income types, non-homothetic preferences, and a deterministic Lucas tree. As in Straub (2019), we find that a non-homothetic taste for wealth can account for the relationship between the saving rate and permanent income observed in the data. We also find that those non-homothetic preferences imply a positive relationship between permanent labor income inequality and the price of financial assets. This model can thus generate large capital gains and an increase in the wealth-to-output ratio without any increase in the aggregate saving rate – which is by construction constant – as it has been observed in the U.S.

Finally, we incorporate this insight into a quantitative two-asset heterogeneous agents model featuring permanent labor income inequality, non-homothetic preferences, and imperfect competition. We calibrate this model on the U.S. economy in the 1960s and compute a transition from 1970 to 2020, where we match the observed increase in labor income inequality. Our model can match both the level of labor income and wealth inequality observed in the U.S. in 1970, generate a Tobin's Q and wealth-to-output level close to those in the data, and match both the speed and the magnitude of the observed increase in wealth inequality, a feature that most heterogeneous agents model have a hard time reproducing.

In our simulations, changes in the permanent component of labor income inequality increase aggregate savings and decrease the returns on the liquid asset, as in Straub (2019).

[2] Net saving rates remove the impact of unrealized capital gains. Gross saving rate includes unrealized in the measure of both income (also called the Haig-Simons income) and savings. Savings in national accounts don't take into account capital gains.

However, due to imperfect competition and distortions coming from firm taxation, the price of financial assets increases by more than the quantity of physical capital. Those valuation effects create some short-term capital gains that maintain the high return on financial assets, and this increase in the equity premium, in turn, has a positive feedback effect on wealth inequality. The *general equilibrium effect* – the change in prices following a shock to the distribution of permanent labor income – accounts for a significant fraction of the increase in wealth inequality. We find that this general equilibrium effect accounts for 18% of the increase in the wealth share of the top 1% from 1970 to 2020.

Related Literature. Our paper adds to the large literature that attempts to study the determinants of the wealth distribution and its dynamics using quantitative models. Early attempts to analytically characterize the distribution of wealth date back to the 50s and the 60s (Champernowne 1953, Vaughan 1979, Laitner 1979, Stiglitz 1969). The development of numerical methods to solve heterogeneous agent models in the 1980s and 1990s (Bewley 1980, Imrohoroğlu 1989, Huggett 1993, Aiyagari 1994) sparked a new generation of papers quantitatively studying the factors determining the distribution of wealth (see De Nardi & Fella (2017) for a review of the literature). This literature found that the inequality in earnings alone is not able to generate the fat tail observed in the distribution of wealth but that models that include random returns or saving rates can match the data (Benhabib et al. 2011, 2015, Xavier 2021).

This paper belongs to a specific subset of this literature that aims to explain not only the long-term, steady-state distribution of wealth but also its dynamics. As found by Gabaix et al. (2016), the random returns or saving rates that create a fat tail at the steady state also generate too slow dynamics compared to the data. They suggest that models that could account for the fast increase in the top wealth share observed in the U.S. should include either a form of "type dependence" (high-savers or high-returns households) or "scale dependence" (returns and saving rates increasing with wealth). Our model follows this second strategy: the non-homothetic taste for wealth implies a positive relationship between wealth and the gross saving rate. At the same time, our portfolio choice captures the fact that rich households invest a higher share of their wealth into risky assets that yield a higher return. This is coherent with part of the empirical literature that has shown that the wealthiest households have both a higher saving rate (Dynan et al. 2004, Fagereng et al. 2019) and higher returns on their wealth (Fagereng et al. 2020, Bach et al. 2020, Garbinti et al. 2021).[3]

Previous works have also studied how changes in asset returns impact the dynamics of wealth inequality. Favilukis (2013) studied the impact of increasing labor income inequality and higher returns on financial assets. In models with entrepreneurs, Gomez et al. (2016) and Cioffi (2021) also find that positive aggregate shocks pushing stock market returns increase wealth inequality. Finally, in a non-micro-founded model estimated on U.S. tax data, Blanchet (2022) finds that the two main drivers of the increase in wealth inequality have been higher saving rates at the top and capital gains. Compared to this literature, our main contribution is to focus on how an increase in labor income inequality can increase the equity premium through capital gains instead of considering higher returns driven by aggregate TFP shocks.

Hubmer et al. (2021) has been one of the most successful models in matching the level and dynamics of wealth inequality, using an exogenous portfolio choice and excess returns along the wealth distribution. They find that the main drivers of wealth inequality have been a change in taxation and a change in asset returns. Our model differs from theirs along a few key dimensions: (1.) they consider an increase in the variance of earnings shocks, while we consider an increase in permanent labor income inequality, (2.) the heterogeneous saving rates come from a time-varying discount factor in their model (type-dependence), while it comes from a non-homothetic taste for wealth in ours (scale dependence). (3.) their portfolio choices and excess returns are exogenous, while they are endogenous outcomes in ours. We thus view our work as an attempt to build on their contribution by endogeneizing the excess return of wealthy households, a key factor for the dynamics of wealth inequality.

We focus on an increase in post-tax *permanent* labor income inequality, as in Straub (2019), and in line with the recent empirical literature (DeBacker et al. 2011, **?**, Guvenen et al. 2017, **?**, 2021). An alternative in the literature is to increase the variance of the persistent and temporary shocks instead. This can have very different effects on the wealth distribution, as found by Hubmer et al. (2021). Indeed, even if idiosyncratic shocks are

[3] In fact, Fagereng et al. (2020) and Bach et al. (2020) show that even within an asset class, wealthier households have higher returns than poorer ones. Our model partially captures this through random idiosyncratic returns shocks on the risky assets.

the primary source of wealth inequality in heterogenous-agents models, they also create a precautionary motive that pushes households at the bottom of the distribution to save more, to reduce the likelihood of being constrained – and hence being out of their Euler equation. This precautionary motive disappears at the top of this distribution, creating a "buffer-stock" behavior (Carroll 1997). In this framework, increasing the variance of the shocks increases the precautionary motive and hence savings at the bottom of the wealth distribution, decreasing wealth inequality.

An increase in labor inequality coming from the permanent component has a very different effect. First, it reduces the total labor income risk for households with lower wages and increases the risk for households at the top of the distribution, who now face a higher risk of total wage income. As shown by (Straub 2019), with standard homothetic preferences, shifts in the distribution of permanent labor income have almost no impact on aggregate savings and returns since richer households are scaled-up versions of poorer ones once they are sufficiently far away from the borrowing constraint. However, this quasi-neutrality of the distribution of permanent labor income can be broken with a non-homothetic taste for wealth. If the marginal utility of consumption decreases faster than the marginal utility of wealth, richer households will have a higher marginal propensity to save out of permanent income shocks than poorer ones, and shifts in the distribution of permanent labor income will imply shifts in aggregate savings.

Our work also builds on Straub (2019), who studies the impact of non-homothetic preferences on household behavior and wealth inequality. He proves that under homothetic preferences, individual consumption scales linearly with permanent income. In a quantitative model, he finds that when accounting for this non-homothetic behavior, the increase in permanent labor income inequality can account for a large part of the increase in wealth inequality and the decrease in real interest rate observed in the U.S. Our work differs from his on two dimensions. First, compared to his analytical result, our main contribution is to show that only a certain type of non-homothetic preferences make the distribution of permanent labor income non-linear with respect to both the distribution of wealth *and* prices. With Stone-Geary preferences, shifts in permanent labor income will increase wealth inequality but not affect prices. We also derive analytically how shifts in permanent income affect the pricing of an asset, depending on the degree of non-homotheticity. Secondly, in our quantitative model, we focus on the impact of labor inequality on asset prices and how asset prices, in turn, affect the distribution of wealth. To do so, we include a more realistic structure of the firm and a portfolio choice on the household's side. The structure of the firm allows us to study realistically the valuation effects coming from shifts in aggregate savings, while the portfolio choice implies that capital gains can shift the wealth distribution.

A key element of our model is the non-homothetic taste for wealth that allows us to match the marginal propensity to save in the data out of permanent income shocks. Carroll (1998) and Carroll (2000) are the first to study how a taste for wealth can explain both the higher saving rates of the rich and the higher share of risky assets in their portfolio. Kumhof et al. (2015) shows in a two-agent model with a non-homothetic taste for wealth that an increase in permanent labor income inequality increases household debt and the endogenous risk of a debt crisis. A non-homothetic taste for wealth has also been used in the New Keynesian literature to explain the zero-lower bound and the impact of secular stagnation (Michau 2018, Mian et al. 2021, Michaillat & Saez 2021).

This paper is organized as follows. In the first section, we summarise some stylized facts about the increase in labor and wealth inequality, the evolution of the wealth-to-output ratio, and the valuations of capital. In the second section, we study the impact of a labor income shock in a two-agent analytical model with non-homothetic preferences and capital gains. The third section presents our quantitative model to measure the size of the general equilibrium channel. The last section presents the main results.

1 Stylized facts

This section documents the main stylized facts about the U.S. that motivate this paper. At the aggregate level, we report (1.) the increase in labor income and wealth inequality, (2.) the rise in the price of capital, and (3.) an increase in the equity premium. At the cross-sectional level, we use the Survey of Consumer Finance (SCF) to document that richer households (1.) have a higher marginal propensity to save and (2.) invest a higher share of their wealth into risky assets.

This paper's main estimates of labor income and wealth inequality are from Piketty et al. (2018), which combines tax, survey, and national accounts data to estimate the increase in the top labor and income shares. The primary advantage of their method is that it matches aggregate values of income and wealth in the national accounts. The values are similar, if a bit lower, to the ones obtained in the SCF (see Figure 1). Both document a significant increase in the top 10% and top 1% in wealth and labor income shares between 1980 and 2020.

Figure 1: Top labor income and wealth shares

Note: This figure shows the evolution of the top 1% and top 10% pre-tax labor income shares and wealth shares from the Distributional National Accounts in Piketty et al. (2018) (PSZ, blue line) and in the Survey of Consumer Finance (SCF, orange line, authors calculations).

The distribution of wealth displays a fatter right tail than the distribution of labor income, which has increased over time. The top 1% income share has risen by 6 p.p. between 1980 and 2020, according to PSZ, against 15 p.p. for the top 1% wealth share. Not only did wealth inequality increase faster than labor income inequality, but the total wealth held by households also increased faster than national income (Figure 2, left panel). In contrast, the personal saving rate remained relatively constant.

It is important to recall that because measures of national income do not account for capital gains, an increase in wealth does not necessarily come from increased net savings from households. Indeed, if we decompose the wealth-to-output ratio between a saving and a capital gain component, we find that almost all of the increase in the wealth-to-output ratio comes from an increase in the price of wealth and not from an increase in savings (see Figure 2, right panel).[4] This increase in the valuation of capital is also clearly visible in the valuation of the firms: the aggregate Tobin's Q, which is the ratio of the market value and the book value of U.S. corporations, went from below 1 before the 1980s to above 1 today.

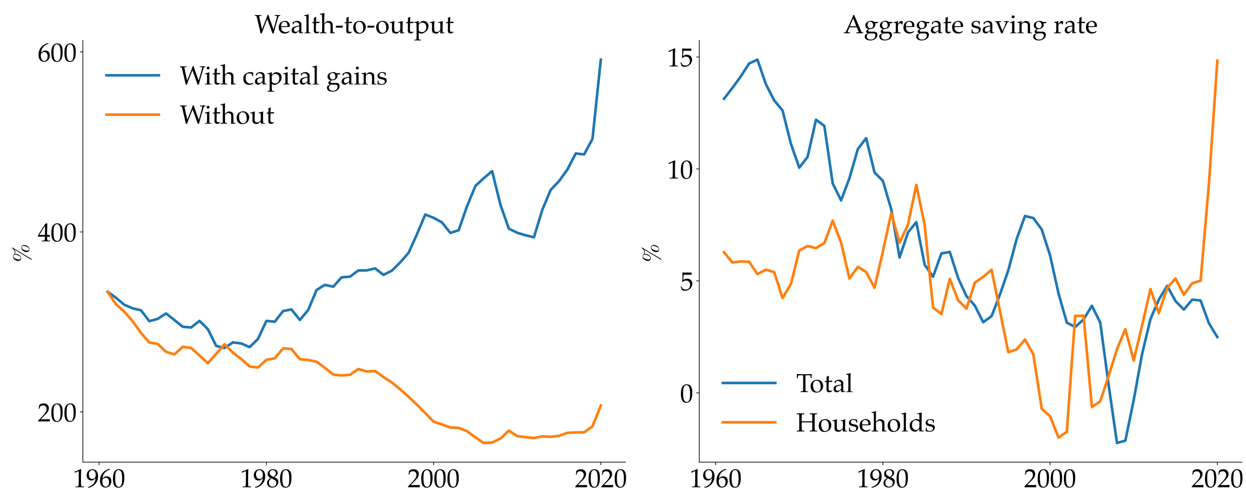

Figure 2: Aggregate household wealth and saving rate

Note: This figure shows the evolution of the household wealth-to-national-output ratio. The decomposition follows Piketty, Zucman (2014) by computing aggregate wealth, removing any valuation effect. This decomposition comes from the following law of motion of wealth $W_{t+1} = W_t + S_t + KG_t$ where W_t is the household aggregate wealth, S_t is aggregate savings from households and KG_t are capital gains. The figure on the right plots the net aggregate saving rate in the NIPA accounts.

Two things should be noted. First, this increase decomposition is not a counterfactual exercise: in the absence of capital gains, households might have increased their savings rate to satisfy their savings needs. It is, however, coherent with Fagereng et al. (2019), who

[4] That is, $\frac{W_t}{GNP_t} = \frac{W_{t-1}+S_{t-1}+KG_{t-1}}{GNP_t}$, where W_t is the total real wealth held by households, S_t is total savings and KG_t is capital gains,

found the higher saving rates from richer households came mostly from capital gains. Secondly, this increase in the valuation of the firms is also due to other factors than the increase in labor income inequality: the decrease of the dividend and the corporate tax rate and the increase in markups also played an important role (Piketty & Zucman 2014, Brun & Gonzalez 2017).

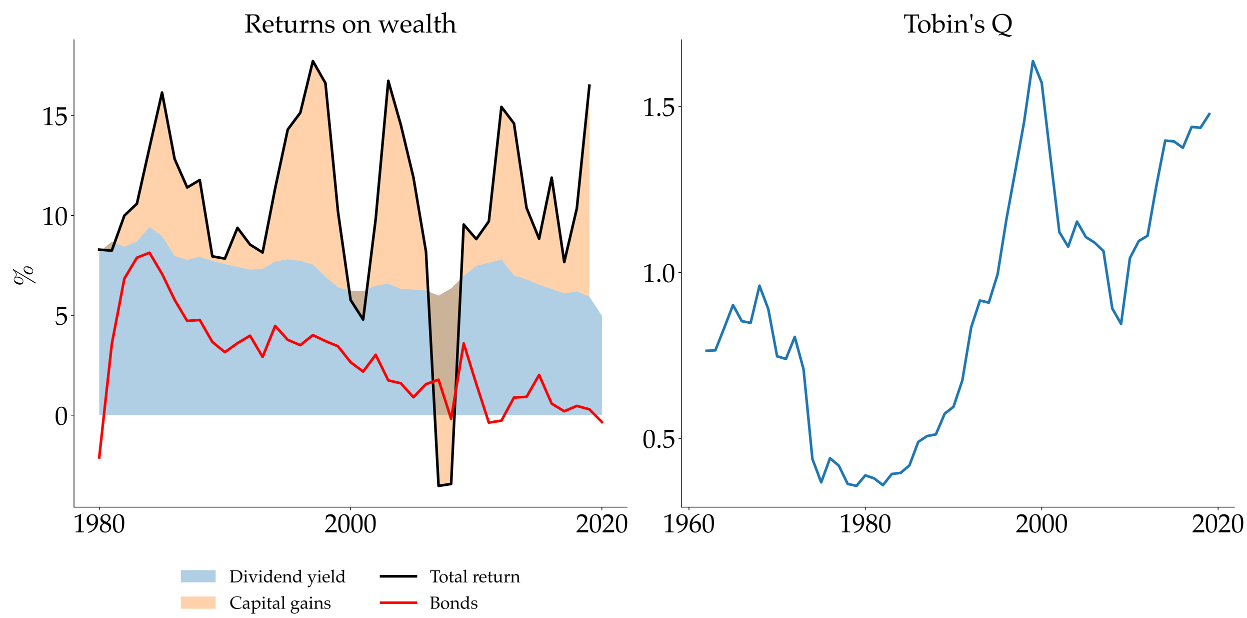

Figure 3: Returns on wealth and Tobin's Q

Note: This figure plots returns on total wealth in the U.S., computed from NIPA, and the Tobin's Q reported in Piketty et al. (2018).

This considerable accumulation of wealth at the aggregate level has been accompanied by an increase in the equity premium (Caballero et al. 2017, Reis 2022). In Figure 3, we plot in black the total real return on national wealth in the U.S., computed from national accounts reported by Piketty & Zucman (2014), against the real return on the 10-year Treasury Bills. Apart from medium-term fluctuations, the total real return on U.S. wealth has remained relatively constant over time while the real return has dramatically decreased. Moreover, capital gains are volatile but represent a significant proportion of the total return.

At the cross-sectional level, two stylized facts should be noted: (1.) the saving rate is an increasing function of permanent income, and (2.) richer households invest a higher share of their wealth into risky assets.

Those two facts are crucial to explaining the primary mechanism in our model: the difference in saving rates along the distribution of permanent income implies that an increase in labor income inequality can affect the desired aggregate savings. Those changes will, in turn, affect the valuation of capital, which will create an excess return on capital compared to the safe asset. Because richer households own a higher share of risky assets, this effect will amplify wealth inequality, creating a feedback loop between labor income and wealth inequality.

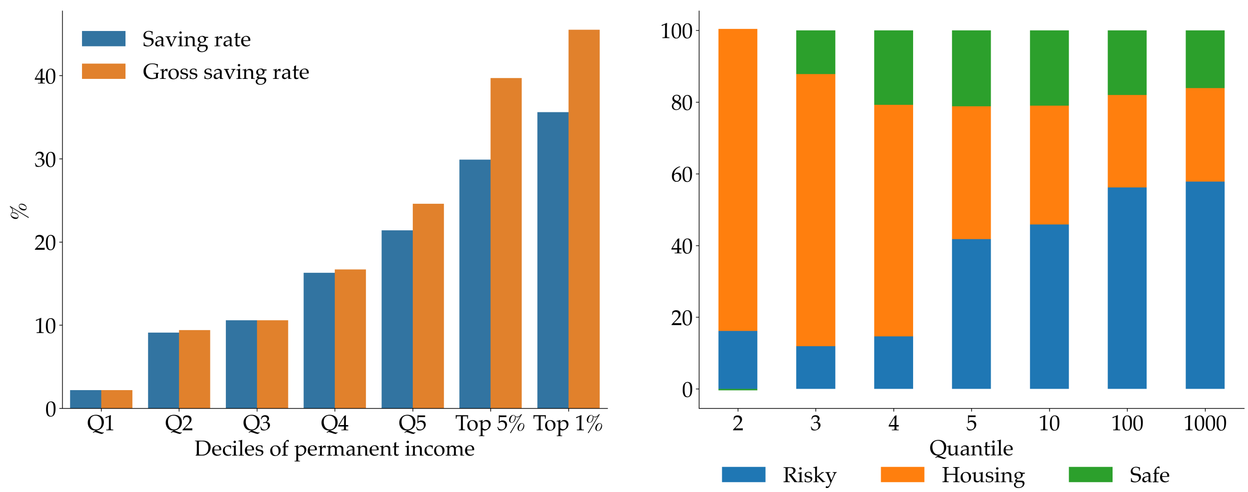

Figure 4: Households and saving rates portfolio in the SCF along the wealth distribution, 1989

Note: The figure on the left reproduces the saving rates computed in the SCF in 1989 by Kumhof et al. (2015). The saving rates are computed with a quantile regression and controlling for age. Gross saving rates account for unrealized capital gains.

2 Analytical model

In this section, we analyze what preferences can account for the positive relationship between increasing marginal propensity to save out of permanent income and permanent income in a simple analytical model with two-agent and a deterministic Lucas tree. We then explore the general equilibrium implications of an increase in permanent labor income inequality.

We show analytically that, under standard preferences for consumption, the marginal propensity to save out of permanent labor income shock is zero. When we add a homothetic taste for wealth, the marginal propensity to save is constant, and the distribution of permanent labor income has thus no impact on the price of equity. Only a particular form of non-homotheticity in the taste for wealth can generate a marginal propensity to save, which is an increasing function of permanent labor income, as we observe in the data. In this case, the distribution of labor income becomes non-neutral and shapes both the wealth distribution and the price of equity. Finally, we show that in a transition from a steady state with low inequality to a steady state with high inequality, capital gains initially increase the rise of the top 1% wealth share by increasing returns to wealth.

2.1 Environment

Both agents $i \in \{1, 2\}$ differ in the level of endowment z_i they receive at each period and in the initial allocation of the deterministic Lucas-tree $s_{i,0}$.[5] Without loss of generality, $z_1 < z_2$, the total endowment is normalized to 1, and we define $z \equiv z_2$ (and so $z_1 = 1 - z$). One unit of Lucas tree delivers one unit of final good at each period.

Taking the sequence of prices for the Lucas-tree $\{q_t\}_{t=0}^{\infty}$ as given, each household i chooses a stream of consumption to maximize lifetime utility, taking into account that she values holding wealth by itself:

$$\max_{\{c_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta_i^t [u(c_{i,t}) + \gamma v(q_t s_{i,t})],$$

subject to the budget constraint $c_{i,t} + q_t s_{i,t+1} = (q_t + 1)s_{i,t} + z_i$ and the non-negativity constraint on equity $s_{i,t+1} \geq 0$. Consumption is CRRA with a risk aversion of σ. For the function $v(.)$, we closely follow Straub (2019) and choose:[6]

$$v(q_t s_t) = \frac{(q_t s_t + \zeta)^{1-\Sigma} - 1}{1 - \Sigma}.$$

This function has two non-homotheticity parameters: the constant term ζ and the relative risk aversion Σ.[7] Both agents can trade equity $s_{i,t+1}$ in t. The equity market must clear at all times:[8]

$$\forall t, s_{1,t+1} + s_{2,t+1} = 1.$$

[5]Endowments should be viewed as permanent labor income. Differences in endowments could reflect differences in productivity between the two agents.

[6]Our functional form only differs from Straub (2019) due to the constant term ζ.

[7]We denote that constant, the Stone-Geary parameter.

[8]Given Walras law, if the equity market clears, the goods market must also clear.

We normalise $s = s_2$ and $s_1 = 1 - s$.

Definition: A competitive equilibrium is an initial distribution of endowment $\{s_{1,0}, s_{2,0}\}$, sequences $\{c_{i,t}\}_{t=0}^{\infty}$, $\{s_{i,t}\}_{t=0}^{\infty}$, and $\{q_t\}_{t=0}^{\infty}$ such that households solve their problems by taking prices as given and the market for shares and the goods market clear at all times.

2.2 Analytical results

We start from the benchmark case with (1.) no taste for wealth $\gamma = 0$ before (2.) adding a homothetic taste for wealth and (3.) the non-homothetic taste for wealth case.

Without a taste for wealth

Proposition 1. *When preferences are homothetic, the price of the equity is*

$$q_t = \left(\frac{1}{\beta} - 1\right)^{-1}.$$

$\forall t$, $s_{1,t} = s_{1,0}$ and $s_{2,t} = s_{2,0}$.

Proof. See Appendix A.1.

Proposition 1 tells us that the equilibrium price for the Lucas tree is such that the two agents are indifferent between saving and dissaving, and their wealth is constant over time. In this setting, the marginal propensity to save out of permanent income is zero $ds/dz = 0$ while consumption reacts one-to-one with variations in permanent income $dc/dz = 1$.

This equilibrium can be thought of as an autarky equilibrium. Because both agents value the asset at exactly the same rate β, they will never have an incentive to trade. We thus have an infinite number of possible steady states, which depends on the initial allocation the two agents start with. We have two ways to move away from this result. The most standard way is to bring in heterogeneous discount factors with $\beta_1 < \beta_2$. The other way is to move away from homothetic preferences with $\gamma > 0$.

Proposition 2. *When discount factors are heterogeneous $\beta_i > \beta_{-i}$, at the steady state, the non-negativity constraint of agent $-i$ is binding, $s_{-i} = 0$, and $q = \left(\frac{1}{\beta_i} - 1\right)^{-1}$.*

Proof. See Appendix A.2.

With heterogeneous discount factors, we now have a unique steady state. The impatient household will always hit the borrowing constraint, and the patient household holds all the wealth in the economy. This result confirms a finding in the literature that heterogeneous discount factors can generate a very high level of wealth inequality (Hubmer et al. 2021). However, this result does not depend on the endowment distribution $\{w_1, w_2\}$ and does not account for the relationship between marginal propensity to save and permanent labor income. In the steady state, we recover that $ds/dz = 0$ and $dc/dz = 1$.

In this case, in the steady state, the level of labor income inequality is neutral on the level of wealth inequality.[9] This is an unappealing result, given the strong correlation between the two in the data. It also generates a marginal propensity to save out of permanent income are constant and equal to 0 for all levels of wealth $ds/dz = 0$.

Adding a taste for wealth

Proposition 3. When $\gamma > 0$, focusing on interior solutions, the price of equity q and the share of the Lucas tree held by agent 2, s, are implicitly defined by the following two equations:[10]

$$\frac{(qs+\zeta)^{-\Sigma}}{(z+s)^{-\sigma}} = \frac{(q(1-s)+\kappa)^{-\Sigma}}{(1-z-1-s)^{-\sigma}}, \tag{1}$$

$$\frac{1}{\beta} - 1 - \frac{1}{q} = \gamma\frac{(qs+\zeta)^{-\Sigma}}{(z+s)^{-\sigma}}. \tag{2}$$

Now, both agents value holding wealth ($\gamma > 0$). Proposition 3 tells us that, at the steady state, the price of the Lucas tree depends on the level of steady-state consumption and wealth.

Equation (1) tells us that, at the equilibrium, the two agents equalize their marginal rate of substitution between the marginal utility from wealth and the marginal utility from consumption. The two agents will trade shares because they give a different value to savings

[9] This result would not hold in an incomplete market setting with idiosyncratic shocks. In this case, the level of buffer stock savings will be proportional to the level of permanent income. However, as shown by Straub (2019), the linearity of the consumption function out of permanent income can be extended to a model with precautionary savings, which implies that $ds/dw > 0$ but constant as a function of permanent income.

[10] Which happens whenever w is not too close from 1.

depending on their permanent income. Wealth is both a way to delay consumption and a good that yields utility. Equation (2) gives that $1 + 1/q < 1/\beta$. The additional motive to accumulate wealth creates a wedge compared to the case without wealth in the utility function where there is no trade.

Proposition 4. *When $\sigma = \Sigma = 1$ and $\zeta = 0$, at the steady state, the value of the Lucas tree and the wealth position are given by:*

$$q = (1 + 2\gamma)(\frac{1}{\beta} - 1)^{-1} \text{ and } s = z.$$

Proof. Appendix A.4.

Assuming log-utility, $\Sigma = \sigma = 1$, and $\zeta = 0$, we can solve analytically for the equity position s and the price of the equity q. A few comments need to be made. First, when $\gamma = 0$, we fall back to the benchmark case with $q = \left(1/\beta - 1\right)^{-1}$. Secondly, the price of the Lucas tree is an increasing function of γ. The more weight the household puts on holding wealth, the higher the equilibrium price to clear the equity market. Thirdly, inequality does not affect the price q because the marginal propensity to save out of permanent income is constant $ds/dz = 1$ for all values of income. This means that households with different levels of permanent income will have the same incentive to hold wealth. Finally, there is a one-to-one relation between wealth inequality and labor income inequality, and the general equilibrium effect neither dampens nor amplifies the direct impact.

Now, we introduce non-homotheticity by allowing for a positive ζ but keeping $\sigma = \Sigma$.[11] We can solve analytically for the value of s as a function of q.

Proposition 5. *When $\sigma = \Sigma$, at the steady state, the quantity of Lucas tree held by agent 2 is given by:*

$$s = \min\left\{\frac{z(q + 2\zeta) - 2\zeta}{q - 2\zeta}, 1\right\}.$$

Proof. See Appendix A.5.

In partial equilibrium, keeping q as constant, the level of s (and of wealth inequality in this setting) is an increasing function of the endowment of agent 2, z. It does not depend

[11] Preferences are now non-homothetic due to the ζ parameter.

on γ. Hence, even if the weight on the taste for wealth is small, it is sufficient to generate a non-trivial wealth distribution.[12] The wealth position of agent 2 is an increasing function of labor income inequality which is not the case with heterogeneous discount factors. We need to check if this remains valid in general equilibrium when q can adjust.

Proposition 6. *When $\Sigma = \sigma = 1$ and $\zeta > 0$, the price of the Lucas tree is given by :*

$$q = \frac{\beta\gamma + \beta\zeta + \frac{\beta}{2} - \zeta + \frac{\sqrt{4\beta^2\gamma^2+8\beta^2\gamma\zeta+4\beta^2\gamma+4\beta^2\zeta^2-4\beta^2\zeta+\beta^2-8\beta\gamma\zeta-8\beta\zeta^2+4\beta\zeta+4\zeta^2}}{2}}{1-\frac{1}{\beta}}.$$

And we have that: $dq/dz = 0$.

Proof. Appendix A.6

Proposition 6 gives us that, in general equilibrium, the marginal propensity to save out of permanent income is positive but constant whatever the level of labor income w:

$$\frac{ds}{dz} = \frac{q+2\zeta}{q-2\zeta} > 0.$$

Even if we already introduce the non-homothetic parameter, ζ, inequality in endowments has no effect on prices. Indeed, when the distribution of permanent income gets more unequal, the dissaving at the bottom is perfectly offset by the increase in saving at the top, keeping aggregate savings equal and everything else equal. Prices thus remain constant. The result of Proposition 6 on the neutrality of the permanent labor income distribution can be extended to the non-log case as long as $\sigma = \Sigma$.

Proposition 7.

$$\text{When } \Sigma < \sigma, \frac{d^2s}{dz^2} > 0, \text{ and } \frac{dq}{dz} > 0,$$

$$\text{When } \Sigma > \sigma, \frac{d^2s}{dz^2} < 0, \text{ and } \frac{dq}{dz} < 0.$$

Proof. See Appendix A.7.

The first part of Proposition 7 tells us that the marginal propensity to save out of permanent income increases with permanent income whenever $\Sigma < \sigma$. In that case, the

[12] Neither given by the initial allocation nor a corner solution like with heterogenous β.

permanent labor income distribution becomes non-neutral on prices. Indeed, when the permanent labor income distribution becomes more unequal, the dissaving at the bottom is more than offset by the increase in saving at the top, so the price of wealth has to increase.

The second part of Proposition 7 tells us that the price of the Lucas tree is an increasing (decreasing) function of inequality in endowments as long as the risk aversion for consumption is greater (smaller) than the risk aversion for wealth. This result is quite intuitive. When $\sigma > \Sigma$, for low-income levels, the marginal utility with respect to consumption is relatively larger than the marginal utility with respect to wealth. The household is better off by increasing consumption compared to increasing wealth.

However, as income increases, the marginal utility with respect to consumption drops faster than the marginal utility with respect to wealth. The household wants to devote a higher share of income to accumulating wealth for higher income levels. The high-income household is willing to accumulate more wealth. In general equilibrium, total wealth must sum up to 1. Any increase in the wealth of the high-income agent must be compensated by a fall in wealth of the low-income agent. Hence, the Lucas tree's price has to increase to push the low-income agent to dis-save and the high-income agent not to over-save. The marginal propensity to save out of permanent income is positive and increases with wealth.

In this model, aggregate savings must be constant and equal to 1, by definition. Thus, changes in the distribution of permanent labor income will increase the *value* of wealth without an increase in aggregate savings. In other words, who owns the wealth will impact the price of wealth. This is a key result: the permanent labor income distribution is non-neutral on prices whenever the marginal propensity to save or consume out of permanent income is not constant. When the MPS out of permanent income is constant, any change in the distribution in permanent income leaves aggregate variables constant.

We summarise what we have seen:

1. With homothetic preferences and homogeneous discount factors, the distribution of endowment is completely neutral: it does not affect wealth distribution and prices. The wealth distribution is indeterminate and is equal to the initial distribution.

2. With Stone-Geary non-homothetic preferences (preference shifter ζ *but* the same risk aversion for consumption and wealth $\sigma = \Sigma$) and homogeneous discount factors, the distribution of the endowment is now *partially* neutral: it affects the wealth distribution, *but* it does not affect prices. The wealth distribution is an interior solution (as long as the borrowing constraint is big enough and inequality is not too high).

3. With non-homothetic preferences (preference shifter ζ *and* different risk aversion for wealth $\sigma \neq \Sigma$), the distribution of endowment is not neutral: it affects the wealth distribution and prices.

The main takeaway from this discussion is that a taste for wealth is consistent with micro evidence of the positive and increasing marginal propensity to save out of permanent income. Those preferences also imply that the distribution of permanent income becomes non-neutral for asset prices. We will explore the quantitative implications it has on the pricing of assets and wealth inequality in the rest of the paper.

2.3 Wealth inequality dynamics

Until now, we have focused on the impact of an increase in permanent labor income inequality on the price of financial assets at the steady state. In this subsection, we focus on the impact of this increase in the price of financial assets on the dynamics of wealth inequality.

As shown in the left panel of Figure 5, an unexpected increase in permanent labor income inequality[13] implies that the long-term price of the financial asset q_t increases. In a transition, this higher long-term price has two main effects on the return on financial assets: (1.) in the long run, the return d/q decreases, (2.) in the short run, the increase in the price creates some short term capital gains, pushing up the return. Indeed, at period 0, households bought the financial asset at the valuation of the initial steady state. However,

[13] In this paper, shocks are always computed as MIT shocks. That is, households unexpectedly start a new period in a new economic environment, in this case, characterized by a higher z_2 and a lower z_1. This shock was not expected by agents, but once it's realized, they expect the correct future path of prices.

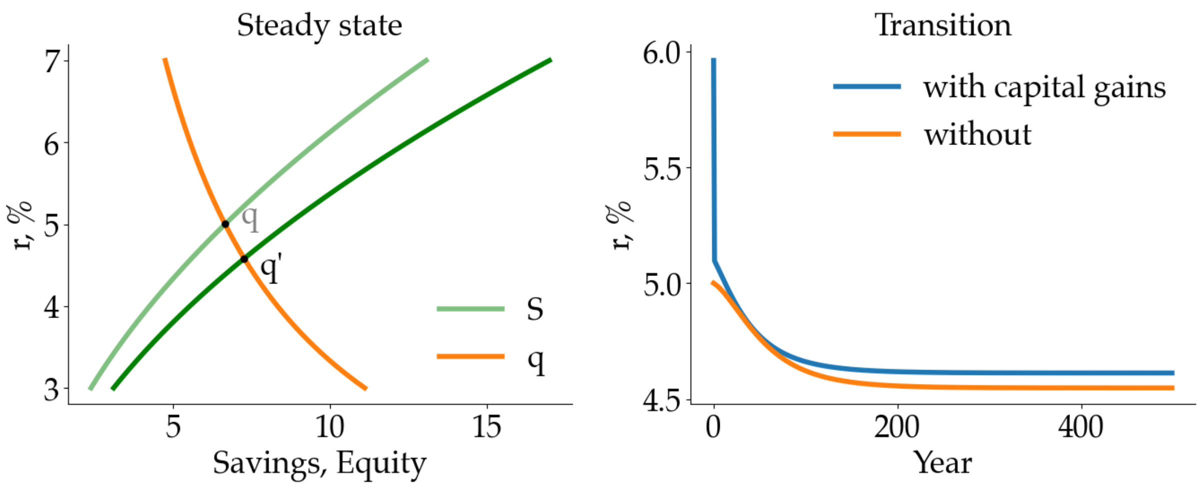

Figure 5: Steady-state equilibrium and evolution of returns following an increase in permanent labor income inequality

Note: The left figure shows the steady state equilibrium on the asset market. The light green curve shows the asset supply from households when inequality is low. The darker green line shows the asset supply when inequality is high. The right panel displays the equilibrium interest rate in a simulated MIT transition between those two steady states in a model with and without capital gains. See Appendix XXX for details.

once the shock is revealed, the price jumps to a higher value, and the return becomes

$$r_0 = \frac{q_1 + d}{q_{ss}} > \frac{d}{q_{ss}}.$$

This one-time increase in the return on financial assets has, in turn, an impact on the distribution of wealth, since it redistributes a share of total income towards the owners of capital. Figure 6 shows the effect of this endogenous change in returns on the top 1% wealth share in three different models of household behavior.

In the three models, the effect follows a similar pattern: it is positive for the first periods of the transition and becomes negative over the long run. This pattern follows the two effects on the return mentioned before in the short run, rich households enjoy unexpected capital gains on their wealth, increasing their financial income and hence their wealth position. Over the long run, however, the effect reverses and the higher valuations decrease the interest rate and their financial income.

Even though the patterns are similar in the two-agent, the one-asset heterogeneous agent model, and the two-assets heterogeneous agent model, the magnitude of the effect varies importantly. In the two-agent model, the general equilibrium effect is small, only 0.4% of

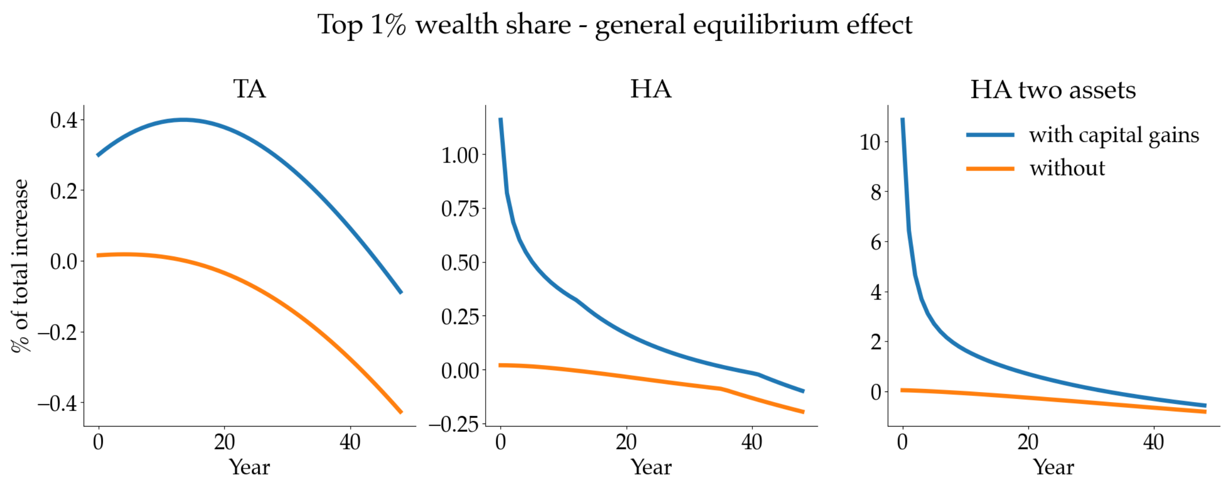

Figure 6: Increase in the top 1% wealth share explained by the endogenous change in returns

Note: This figure shows the general equilibrium effect on the top 1% wealth share in three different models: the two-agent models (left panel), a simple heterogeneous-agent model (center), and a two-asset heterogeneous agent model (right). See Appendix B for more details about the HA and HA-two asset models.

the total increase in the 1% wealth share at the peak. In contrast, the general equilibrium effect is two times larger in the one-asset heterogeneous agent model, reflecting the higher degree of wealth inequality. Indeed, when the steady-state top 1% wealth share is higher, an increase in financial returns accrues mostly to rich households, amplifying the effect on wealth inequality. The two-asset structure amplifies this effect further by allowing households to endogenously invest a higher share of their wealth into the illiquid asset. Since capital gains are paid only to the illiquid account, the distribution of capital gains is even more unequal than in the heterogeneous-agent model. In this setup, the general equilibrium effect is large: more than 10% of the total increase in the 1% wealth share is explained by the endogenous change in return.

This exercise shows that our analytical results on the non-neutrality of the distribution of permanent income on the price of financial assets are fairly general and still hold when we add idiosyncratic productivity shocks and a two-asset structure. However, those settings have a large impact on the magnitude of the general equilibrium effect on the top 1% wealth share, as they amplify the share of capital gains going to the top 1%. Our quantitative model in the next section will build on this insight, using a more realistic supply side.

To understand the specificity of the capital gains effect, we compare the general equilibrium effect in this baseline model to a model that allows agents to accumulate physical capital and to produce the consumption goods using a Cobb-Douglas production function[14]. In this model, the relative price of capital compared to the consumption good is one, and capital gains are thus absent by construction. The orange line in Figures 6 and 5 display the equilibrium return and the general equilibrium effect in this model. We abstract from depreciation, and the returns are thus given by

$$r_t = \alpha Z k_t^{\alpha-1}.$$

In this economy, an increase in savings has a "trickle-down" impact, as in most neoclassical models. The higher savings from households increases the stock of physical capital, which increases output and decreases returns. Since the price of physical capital is set to one compared to the consumption good, the transition never features capital gain and the general equilibrium effect is strictly negative. The higher accumulation of capital decreases the financial income of the rich, limiting their accumulation of wealth.

3 Quantitative model

Our analytical model suggests that changes in the distribution of permanent labor income can strongly impact asset prices if we account for the relationship between saving rates and permanent income. We propose a model incorporating this insight into an Aiyagari-style economy with two assets, imperfect competition, and a permanent income distribution. There is no aggregate TFP risk, and aggregate shocks on the distribution of permanent labor income are modeled as MIT shocks[15].

[14] See Appendix XXX for more details on the calibration and the simulations of this economy.

[15] A MIT shock is a one-time unexpected shock on some parameter. From the model point of view, this shock is a zero-probability event, and the following transition towards the steady state is completely deterministic. Thus households don't have rational expectations with respect to the variable that is shocked, in this case, the distribution of permanent income.

3.1 Setup

Households

Demographic structure. There is a continuum of the household of mass one. Each household supplies inelastically one unit of labor with a level of permanent productivity z_i, and has a probability of dying ξ at each period[16]. Every household is also subject to persistent, idiosyncratic productivity shocks. The productivity z_t follows a log AR(1) process:

$$\log(e_t) = \rho \log(e_{t-1}) + \varepsilon_t^e \text{ with } \varepsilon_t^e \sim \mathcal{N}(0, \sigma^e).$$

Households can insure against the risk of dying with a non-optimal level of wealth by buying an insurance contract on an annuity market, as in Yaari (1965) and Blanchard (1985). When a household dies, its wealth is redistributed to surviving households by the annuity market, and another household is born with zero wealth to maintain a constant population.

Portfolios. To insure against idiosyncratic shocks and satisfy their taste for wealth, households can invest in two saving instruments with different degrees of liquidity. They can invest in a liquid asset b_{t+1} that yields a return r_t^b, or an illiquid asset a_{t+1} that yields a return r_t^a. Accumulating or decumulating illiquid assets is subject to a convex adjustment cost

$$\chi\left(a_{t+1}, a_t\right) = \frac{\chi_1}{\chi_2} \left| \frac{a_{t+1} - \left(1 + r_t^a\right) a_t}{\left(1 + r_t^a\right) a_t + \chi_0} \right|^{\chi_2} \left[\left(1 + r_t^a\right) a_t + \chi_0\right].$$

Households also face a borrowing constraint $b_{t+1} \geq \bar{b}$ on the safe asset, and risky assets cannot be borrowed $a_{t+1} \geq 0$.

Preferences. Households get utility from consumption c_t and from their total wealth level $a_t + b_t$. They have non-homothetic preferences, and the value function is given by:

$$V(a_t, b_t, e_t, z_i) = \max_{a_{t+1}, b_{t+1}} \{u(c_t) + v(a_t + b_t) + (1 - \xi)\beta \mathbb{E}_e V(a_{t+1}, b_{t+1}, e_{t+1}, z_i)\}$$

subject to $c_t + a_{t+1} + b_{t+1} + \chi(a_{t+1}, a_t) = (1 + r_t^a)a_t + (1 + r_t^b)b_t + (1 - \tau_t)(z_i e_t w_t)^{1-\theta}$,

where $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ and $v(a_t + b_t) = \frac{(a_t + b_t + \zeta)^{1-\Sigma}}{1-\Sigma}$.

[16] l stands for low, m for medium, and h for high.

Government

The government finances its exogenous spending G_t and its debt repayment $r_t^b D_t$ by taxing four types of income: (1) a progressive tax on labor income, defined by the parameters $\tau_{t,l}$ and λ, as in HSV, (2) a tax on corporate revenues τ_c, (3) a tax on dividends τ_d, (4) a tax on capital gains τ_g. The law of motion of the public debt is :

$$D_{t+1} = D_t(1 + r_t^b) + G - \Gamma_t - \tau_d d_t - \tau_c(Y_t - Lw_t - \delta K_t) - \tau_g(p_{t+1} - p_t),$$

with $\Gamma_t = Lw_t - \sum_i (1 - \tau_l) \int (w_t e_t z_i)^{1-\lambda} d\mu_t$. We take $\tau_c, \tau_{t,d}, \tau_g$ and τ_l as exogenous. At the steady state, G adjusts to maintain a balanced budget. In the transition, the government follows a fiscal rule[17]

$$G_{ss} - G_{l,t} = \phi(D_{t+1} - D_{ss}).$$

Firms

The supply side is composed of an intermediate-good sector in imperfect competition and final-good producers in perfect competition. The capital stock is owned by the firms in the intermediate sector.

Final-good producer. The representative final-good producer aggregates a continuum of intermediate inputs indexed by j:

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

The price of the final good is normalized to one. The problem of the representative final-good producer is given by:

$$\max_{\{y_{j,t}\}_{i\in[0,1]}} \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} - \int_0^1 p_{j,t} y_{j,t} di.$$

Taking the first-order condition with respect to a variety $y_{j,t}$, we get the demand for each

[17]This fiscal rule is used only for computational purposes to help clear the liquid market during the transition. It does not affect our core results.

intermediary input j:

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t.$$

With $P_t = \left(\int_0^1 p_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$ being the price index. As the representative final-good producer is in perfect competition, the price of the final good will be equal to the marginal cost, P_t. We normalize the price of the consumption good to 1.

Intermediate producers. The firms in the intermediate goods sector produce those inputs using Cobb-Douglas technology, make investment decisions, and pay a corporate tax, a dividend tax, and a capital gains tax to the government. They pay a dividend

$$d_{j,t} = (1-\tau_c)\Big(y_{j,t} - \sum_i w_{i,t} l_{i,t}\Big) - k_{j,t+1} + (1-\delta)k_{j,t} + \tau_c \delta k_{j,t},$$

where

$$y_{j,t} = k_{j,t}^{\alpha} l_{j,t}^{1-\alpha}.$$

They freely set their prices to maximize intertemporal profits:

$$V(k_t, q_{t-1}) = \max_{k_{t+1}, q_t, l_t} (1-\tau_d)d_t + \tau_g q_{t-1} + (1-\tau_g)\kappa(r_{t+1}^a)\frac{V_{t+1}(k_{t+1}, q_t)}{1+r_{t+1}^a},$$

subject to the technology constraint and the demand function of the final-good producer:

$$y_{j,t} = p_{j,t}{}^{-\varepsilon} Y_t.$$

The $\kappa(r_{t+1}^a)$ in the Bellman equation of the firm creates a wedge between the discount factors of the owners of the firm and the discount factor that the managers of the firm will use to determine future investment in physical capital. We use this friction to match the aggregate Tobin's Q in 1970, as in Brun & Gonzalez (2017). The following subsection explains how this friction and the corporate and dividend taxes affect the equity and physical capital schedule.

The intermediate producer chooses labor and investment to maximize the firm's value.

This yields the following first-order conditions:

$$\frac{1+\frac{r_t^a}{1-\tau_g}}{\kappa(r_t^a)} = \alpha(1-\tau_c)\frac{\varepsilon-1}{\varepsilon}k_{j,t}^{\alpha-1}l_{j,t}^{1-\alpha} + 1 - (1-\tau_c)\delta,$$
$$w_t = (1-\alpha)\frac{\varepsilon-1}{\varepsilon}\frac{y_{j,t}}{l_{j,t}}.$$

Since we assume that all intermediary firms are the same in our economy, that the aggregate labor supply is equal to 1, and that there is no price rigidity, $y_{j,t} = y_t$, total output is given by:

$$Y_t = \left(\int_0^1 y_t^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} = y_t = k_t^\alpha$$

Illiquid asset market In the illiquid asset market, in period t, equity shares are traded at a price p_{t+1}. The return on the illiquid asset is given by:

$$1 + r_{t+1}^a \equiv \frac{(1-\tau_d)d_{t+1} + q_{t+1} - \tau_g(q_{t+1} - q_t)}{q_t}.$$

Using this relationship, the price of the firm can be expressed as a function of dividends and returns, assuming no bubble:

$$q_t = \sum_{j=1}^{\infty}\left(\prod_{i=0}^{j-1}\frac{1}{1+\frac{r_{t+1+i}^a}{1-\tau_g}}\right)\frac{1-\tau_d}{1-\tau_g}d_{t+j}.$$

Equilibrium

Let $\mathcal{A}$ be the space for illiquid wealth, $\mathcal{B}$ be the space for liquid wealth, $\mathcal{Z}$ the space for productivity, and $\mathcal{S}$ the space for permanent type.

An equilibrium in this economy is defined as paths for household and firm decisions $\{a_{t+1}, b_{t+1}, k_{t+1}, c_t, d_t, l_t\}_{t=0}^\infty$, factor prices $\{r_t^a, r_t^b, \{w_{i,t}\}_{i\in\mathcal{S}}\}_{t=0}^\infty$, the tax policy $\{\lambda_t, \tau_{l,t}, \tau_{t,c}, \tau_{t,d}, \tau_{t,g}\}_{t=0}^\infty$, measures $\{\mu_t\}_{t=0}^\infty$, and aggregate quantities, such that, for all t: (1) households and firms solve their objective functions, (2) the sequence of distributions satisfies aggregate consistency conditions, (3) the government budget and decision rue constraint holds, and (4) all markets clear at all times:

1. The liquid asset market clears:

$$\int_{A\times B\times Z\times \mathcal{S}} bd\mu_t(a,b,z,i) = D.$$

2. The illiquid asset market clears:

$$\int_{A\times B\times Z\times \mathcal{S}} ad\mu_t(a,b,z,i) = q_{t-1}.$$

3. The labor market clears:

$$l_t = 1.$$

4. The goods market clears:

$$Y_t + (1-\delta)K_t = C_t + K_{t+1} + \int_{A\times B\times Z\times \mathcal{S}} \chi(a_{t+1},a_t)d\mu_t(a,b,z,i).$$

3.2 Labor income inequalities and Tobin's Q

Valuation effects under imperfect competition

A key result of this paper is that in the presence of non-homothetic preferences and with imperfect competition, an increase in permanent labor income inequality will increase the Tobin's Q. In this section, we focus on the steady change for analytical clarity.

In the previous section, we studied two polar examples of the supply side. First, a model with a deterministic Lucas tree, where the asset supply is constant and hence all changes in asset demand from households imply an equivalent change in asset prices. This was a pure model of *valuation effects*. Then, we studied a model in which the price of financial assets is set to one and where all changes in asset demand from households imply an equivalent change in *physical quantities*. The imperfect competition setting studied in this quantitative model is an intermediate case, where changes in asset demand from households imply a change in both the price of financial assets and the quantities of physical capital. However, the price of capital reacts more to changes in asset demand than the quantity of physical capital, which means that an increase in asset demand increases the average Tobin's Q, defined as the value of the firm divided by the value of its physical capital.

The reason for this valuation effect is intuitive. With imperfect competition, the valuation of the firm at the steady state can be decomposed in two terms: the net discounted normal profits, and the net discounted excess profits stemming from the pricing power of the firm. A decrease in the real return on illiquid assets will increase the firm's valuation because excess profits will now be discounted at a lower rate. Indeed, the price of equity at the steady state can be written as[18]

$$\begin{aligned} q &= (1-\tau_d)\frac{d}{r^a} \\ &= \underbrace{\frac{1-\tau_d}{1-\tau_g}\frac{\overline{\kappa} r^a K}{r^a}}_{\substack{\text{Net discounted} \\ \textit{normal} \text{ profits}}} + \underbrace{(1-\tau_c)(1-\tau_d)\frac{Y}{r^a \varepsilon}}_{\substack{\text{Net discounted} \\ \textit{excess} \text{ profits}}}. \end{aligned}$$

Hence, when r^a falls, the firm's valuation q increases because of two effects:

$$\frac{dq}{dr^a} = \frac{1-\tau_d}{1-\tau_g}\overline{\kappa}\frac{dK}{dr^a} + \frac{(1-\tau_c)(1-\tau_d)}{\varepsilon}\frac{d(Y/r)}{dr^a}.$$

First, the fall in r^a increases the demand for capital K, and so, the value of the firm. Secondly, the fall in r^a increases the discounted excess profit from the imperfect market structure $\frac{Y}{r^a \varepsilon}$. Due to the second effect, the firm's valuation overreacts to variations in the return on the illiquid market compared to the demand for capital:

$$\left|\frac{dq}{dr^a}\right| > \left|\frac{dK}{dr^a}\right|.$$

Therefore, the average Tobin's Q increases whenever the return on the illiquid market r^a falls:

$$\begin{aligned} Q &= \frac{q}{K} \\ &= \frac{1-\tau_d}{1-\tau_g}\overline{\kappa} + (1-\tau_c)(1-\tau_d)\frac{Y}{rK\varepsilon} \quad \text{and} \quad \frac{dQ}{dr^a} < 0. \end{aligned}$$

Figure 7 summarizes those findings by representing the equilibrium in the illiquid asset

[18]Here, we assume that the friction takes the form $\kappa(r_t^a) = \frac{1+\frac{r_t^a}{1-\tau_g}}{1+\overline{\kappa}\frac{r_t^a}{1-\tau_g}}$. This is to ensure that the capital and price schedules are monotonic. In our calibration, $\overline{\kappa} < 1$.

market. The green line shows the supply of savings from households, while the blue line shows the value of the firm and the orange line shows the capital stock of the firm. The equilibrium is at the intersection of the supply of savings from households and the value of the firm. On the household side, as shown in Section 2, any increase in permanent labor income inequality will increase aggregate savings due to the increasing marginal propensity to save out of permanent income. The dissaving at the bottom is more than offset by the increase in savings at the top, and the supply of savings is shifted to the right. In general equilibrium, r^a should thus decrease. On the firm's side, the imperfect competition setting implies that the price of the firm will react more to changes in returns than its stock of capital, as in Brun & Gonzalez (2017). Thus, when we shift the saving curve upward, as in Figure 7, the value of the firm will increase by more than the stock of capital, increasing Tobin's Q.

Note that the key element in this mechanism is imperfect competition: the price of the firm overreacts to changes in returns because of the increase in net discounted excess profits. The taxes on dividends, capital gains, and corporate revenues, along with the friction on the discount factor of the firm, are not enough to create variations in Tobin's Q when the return on the illiquid asset changes. Indeed, in the perfect competition case, when $\varepsilon \to \infty$, we have

$$Q = \frac{1-\tau_d}{1-\tau_g}\overline{\kappa} \quad \text{and} \quad \frac{dQ}{dr^a} = 0.$$

However, they allow us to match the Tobin's Q observed in the data before 1970.

Capital gains in the transition

In the previous subsection, we showed how an increase in labor income inequality impacts the firm's valuation at the steady state. In this section, we explain how this inequality shock creates some short-term capital gains.

As we showed in the analytical model, an unexpected shock in the distribution of permanent labor income – i.e., a shock on (z_h, z_m) – increases desired aggregate savings. This increase in aggregate savings decreases the long-term real returns (r^a, r^b). As households are surprised by the change in the path of variables, the ex-ante anticipated equity price does not equal the ex-post equity price, increasing the realized return on illiquid assets in

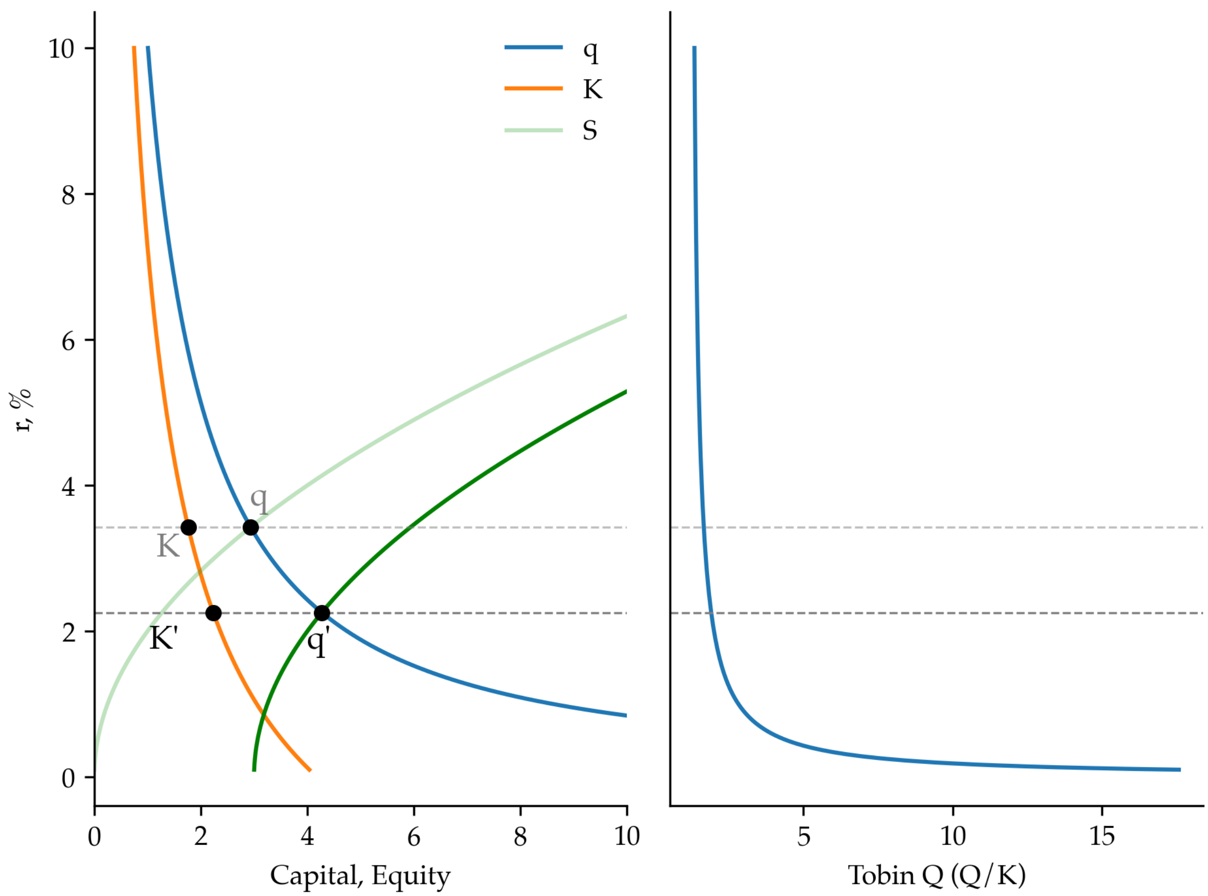

Figure 7: Impact of an inequality shock

Note: This figure shows the equilibrium on the illiquid market at the steady state. The blue line shows the value of the firm as a function of the return r^a, the orange line shows the capital stock of the firm and the green lines show the saving curve of households.

period 0 of the transition. The expected return at the steady state was

$$r^a_{ss} = \frac{(1-\tau_c)d_{ss}}{q_{ss}},$$

but the realized return is

$$r^a_0 = \frac{(1-\tau_c)d_{ss} + (1-\tau_g)(q_1 - q_{ss})}{q_{ss}} > r^a_{ss} \quad \text{if} \quad q_1 > q_{ss},$$

where q_1 is the price at the first period of the transition. Only this initial return on the illiquid asset following a permanent labor income shock differs from the expected return, and there is perfect foresight for the remaining periods of the transition. However, because we model the increase in permanent labor income inequality as a sequence of unexpected MIT shocks between 1980 and 2020, households are consistently surprised by the higher return on the illiquid asset for the initial 50 years of the transition.

This myopic behavior of households in the transition is obviously a strong simplifying assumption, but we argue that it captures some realistic features of household behavior that have been recently developed in works that include some behavioral frictions in heterogeneous models. Our model can be seen as a reduced-form implementation of the behavioral friction introduced by Auclert et al. (2021). In their model, households infrequently update their information sets about aggregate shocks and the price of illiquid assets.

An alternative would be to assume that households had perfect knowledge of the future increase in permanent labor income in 1980. This would be unrealistic, as it implies that households forecasted the entire evolution of the distribution of permanent labor income accurately to predict changes in returns, whereas economists themselves have only recently understood this increase in inequality. Another counterfactual implication of this alternative is that since rich households expect their wage to increase in the future, they would decrease savings today to smooth their consumption, whereas poor households would do the opposite. This consumption smoothing behavior would imply that an increase in permanent labor income inequality decreases wealth inequality.

A last possibility is to compute a one-time shock on permanent labor income inequality. The capital gains would thus be concentrated on the first period of the transition, which speeds up the increase in wealth inequality. This solution has the benefit of being orders of magnitude faster computationally than our benchmark results but doesn't change the main results that we describe in the next subsection. The main results of this one-time-shock transition can be found in the Appendix.

3.3 Calibration

We calibrate our initial steady state on U.S. data between 1960 and 1970.

Permanent income distribution. We calibrate the distribution of permanent labor income inequality, characterized by the parameters $\{z_l, z_m, z_h\}$ on the labor income share observed in the U.S., as measured by Piketty (2018). To move from the empirical labor share to the parameters z_s, we use the following formula

$$z_s = \frac{\omega_s}{\mu_s},$$

where ω_s is the labor income share observed in the data and μ_s is the mass of agents with permanent income z_s in the model.

Households. As standard in the literature, we fix $\sigma = 2$ and internally calibrate the rest of the preference parameters of the household to jointly match the wealth distribution in 1970 and the marginal propensity to save computed on the SCF in 1983 (see Appendix for details). We calibrate the illiquidity parameters on the portfolios observed in the SCF in 1989. We use the year 1989 as it is the oldest vintage available in the SCF with detailed information about portfolio composition. For the idiosyncratic productivity shocks, we follow Straub (2019) and set $\rho = 0.9$ and $\sigma_e = 0.2$.

Table 1: Wealth distribution at the initial steady state

	Bottom 50%	Middle 40%	Top 10%	Top 1%	Top 0.1%	Top 0.01%
Data PSZ (1970)	1	29	69	26	9	
Model	3	29	67	27	9	

Government. We fix $(\tau_{1980,c}, \tau_{1980,d}, \tau_{1980,g})$ to $(0.35, 0.4, 0.19)$, as reported by Brun & Gonzalez (2017), and we adjust B to match an aggregate ratio $\frac{B}{q+B} = 30\%$ to match the illiquid-to-liquid ratio reported in the national accounts. λ_{1980}, the progressivity parameter in the

HSV tax function is equal to 0.18, as in Ferriere et al. (2023). We then set $\frac{G}{Y} = 10\%$ and use the labor tax rate $\tau_{l,t}$ to solve the budget constraint of the government.

Table 2: Share of illiquid assets along the wealth distribution

	Q1	Q2	Q3	Q4	Q5	D10	P100
Data SCF (1989)	31	58	71	71	78	79	83
Model	10	39	45	55	76	77	91

Firms. We target a labor share of $\frac{2}{3}$, as is standard in the literature, and fix the elasticity of substitution between inputs $\varepsilon = 6$ to obtain a markup of 20% and adjust the capital share α to match a labor share of $\frac{1}{3}$. We adjust the depreciation rate δ to obtain a ratio $\frac{K}{Y} = 250\%$. We then adjust the firm's patience κ to obtain a Tobin's Q of 0.7.

Table 3: Marginal propensity to save along the permanent labor income distribution

	MPS low	MPS mid	MPS high	MPS Agg
SCF (1983)	0.28	0.45	0.68	0.3
Model	0.25	0.46	0.52	0.27

Table 1, Table 3, and Table 2 report the wealth distribution, the marginal propensity to save along the distribution of permanent labor income, and the portfolio shares at the initial steady state. We use the SCF in 1983 to estimate the marginal propensity to save out of permanent income since it is the only year where the SCF had a true panel[19]. Our model fits well the very top of the wealth distribution, up to the 0.1% share, but slightly underestimates the top 10% wealth share and overestimates the share of the bottom 50%. Our calibration also reproduces the stylized facts of the marginal propensity to save along the distribution of permanent income, and we obtain an aggregate MPS of around 0.3, a calibration similar to the one of Straub (2019). Finally, our portfolio shares match the increasing nature of illiquid assets along the distribution of wealth but underestimate the share of illiquid assets owned by the middle of the distribution.

[19]We follow Kumhof et al. (2015) to estimate the marginal propensity to save, using the same dataset. Note that a panel is also available for the SCF in 2008, but given the specificity of this year in the U.S., we choose not to use it to calibrate our model.

Table 4: Calibration of the model

Parameters	Description	Value	Source
Preferences			
β	Discount rate	0.92	Internally calibrated
γ	Weight on taste for wealth	2	Internally calibrated
σ	CRRA coefficient for consumption	2	
Σ	CRRA coefficient for wealth	1.7	Internally calibrated
ζ	Stone-Geary parameter for wealth	6	Internally calibrated
Illiquidity cost			
χ_0		0.8	Internally calibrated
χ_1		3	Internally calibrated
χ_2		2	Internally calibrated
Production			
α	Capital share	0.2	Capital share of 0.33
z_s	Permanent labor income		Piketty et al (2018)
ε	Inverse markup	6	
δ	Depreciation rate	0.07	Capital-output ratio of 2
$\overline{\kappa}$	Friction in the firm's discount rate	0.1	Target a Tobin's Q of 0.5
labor income process			
ρ	Auto-correlation of idiosyncratic shocks	0.9	Straub (2019)
σ_e	Variance of idiosyncratic shocks	0.2	Straub (2019)
Government			
D	Government debt	0.9	Internally calibrated
ϕ_τ	Elasticity of public debt	0.1	
λ_{1980}	Tax progressivity	0.17	Ferriere et al (2018)
τ_l	Labor income tax	0.12	
$\tau_{1980,c}$	Corporate income tax, 1980	0.35	Brun, Gonzales (2017)
$\tau_{1980,d}$	Dividend tax, 1980	0.4	Brun, Gonzales (2017)
$\tau_{1980,g}$	Capital gains tax, 1980	0.19	Brun, Gonzales (2017)

4 Impact of an increase in permanent labor income inequality

As shown in the previous section, our quantitative model captures the key relationships between labor income inequality, the marginal propensity to save, and households' portfolios while generating a realistic distribution of wealth. We now ask two questions. First, what is the role of each element in our model in determining the distribution of wealth at the steady state? Secondly, how and through which channel did the increase in post-tax permanent labor income inequality affect the distribution of wealth? We answer these questions by (1.) proposing a simple decomposition of the distribution of wealth at the steady state, as in Hubmer et al. (2021), and (2.) by running a transition matching the observed increase in labor income inequality in the data in partial equilibrium and general equilibrium.

4.1 Steady-state decomposition

Table 5: Contribution of different channels for steady state inequality

#		Bottom 50%	Middle 40%	Top 10%	Top 1%
1	No permanent labor income inequality ($z_s = 1$)	5	18	-23	-16
2	No transitory shocks ($\sigma_e = 0$)	-1	-7	7	5
3	No tax redistribution ($\lambda = 0$)	-1	-6	7	4
4	One-asset ($\chi_1 = 0$)	0	2	-2	-3
5	Perfect competition ($\varepsilon = \infty$)	5	9	-15	-14
6	No capital tax ($\tau_g = 0$)	0	-1	1	6
7	No labor tax ($\tau_l = 0$)	1	6	-7	-6

Note: This table displays the p.p. change in the wealth distribution for each quantile after removing a given feature of the model, keeping the calibration constant. For example, the first row says that removing heterogeneity in permanent labor income decreases the top 1% wealth share by 16 p.p.

Table 5 decomposes the factors behind the distribution of wealth in our full model. The main drivers of inequality at the steady state are the distribution of permanent labor income (line 1), the imperfect competition setting (line 5), and the tax on capital (line 6). Note that because of the non-linear nature of the model, the different effects don't sum to the total wealth shares.

The major role of the distribution of permanent labor income is not surprising. As we have seen in Section 2, with a non-homothetic taste for wealth, households with higher levels of permanent labor income have a higher marginal propensity to save and will thus accumulate more wealth than poorer ones. In contrast, the idiosyncratic transitory shocks have a smaller impact on the distribution of wealth, which explains why standard heterogeneous-agent models usually have a hard time generating high degrees of wealth inequality. As in Hubmer et al. (2021), the idiosyncratic transitory productivity shocks *dampen* wealth inequality due to the precautionary motive.

Secondly, the imperfect competition setting also generates a high level of wealth inequality. In our simulations, we set ε to a very high value and adjust α to keep the labor share constant. Thus, the impact of switching to perfect competition displayed here does not come from a change in the labor share but from two different effects on the firm side: an increase in the demand for capital of the firm and a decrease in the equilibrium profits and valuation of the firm. Those two effects increase the capital stock in equilibrium and, hence, the total output, increasing wages and decreasing wealth inequality.

Thirdly, removing the taxes on dividends, capital gains, and corporate revenues also decreases wealth inequality in equilibrium. This comes from two effects. First, for a given level of physical capital, removing those taxes increases dividends and, thus, the return on equity, benefiting households already owning a lot of wealth. Secondly, removing those taxes also changes the firm's demand for capital and its valuation in equilibrium. In our calibration, this increases the Tobin's Q and decreases the quantity of physical capital in the economy, decreasing output and wages and amplifying wealth inequality.

Finally, the two-asset structure endogenously creates the type of "scale-dependence" described by Gabaix et al. (2016); that is, the return becomes an increasing function of wealth. Indeed, due to the transaction cost associated with illiquid assets, portfolios are not homogenous across the wealth distribution. At the bottom of the wealth distribution, the probability of hitting the borrowing constraint makes households risk averse, and, as a result, poor households prefer to hold liquid assets. As households get richer, the probability of hitting the constraint falls, and the fraction of risky assets in their portfolio increases, increasing the total returns they enjoy on their wealth. This mechanism remains relatively small since the illiquidity premium is small in our model. It is, however, important in determining the strength of the general equilibrium effect, as we will show in the next section.

4.2 Transition

We now focus on the main quantitative exercise of this paper: the simulation of this economy following a shock on the distribution of permanent labor income. More precisely, we simulate a sequence of unexpected MIT shocks on the sequence of (z_h, z_m, z_l) and on the tax progressivity parameter λ_t, using the Sequence Space Jacobian method (Auclert et al. 2021). A description of the algorithm is available in the Appendix. We set the $(z_{l,t}, z_{m,t}, z_{h,t})$ to the pre-tax labor income shares observed in the data in Piketty et al. (2018) from 1970 to 2020. As shown in Figure 8, we have a perfect match for the top 10% and top 1% labor income shares since there is a one-to-one mapping between those empirical targets and our parameters. For the progressivity parameter λ_t, we use estimates from Ferriere & Navarro (2018).

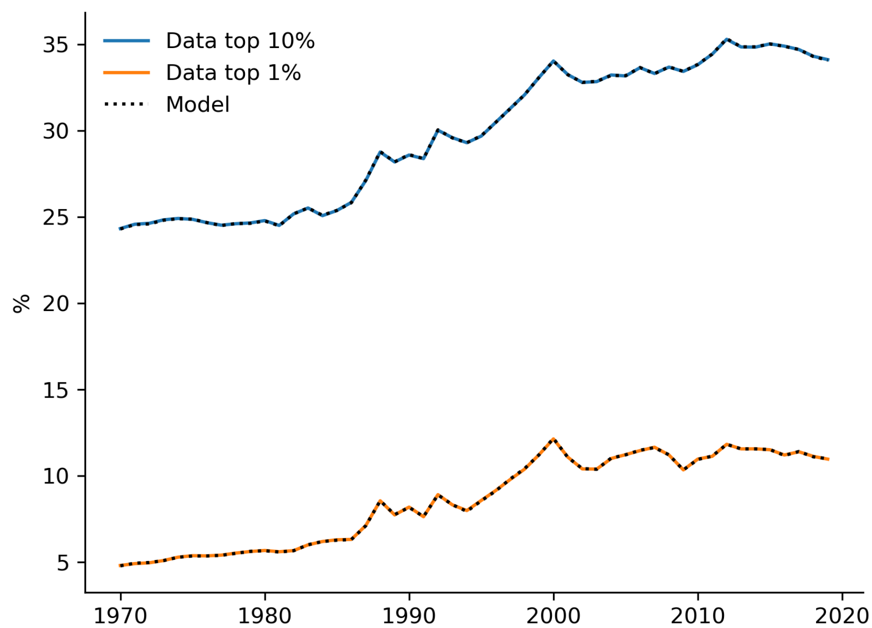

Figure 8: Top labor income shares in the data and in the model

Note: This figure shows the top pre-tax labor income shares in the data (Piketty et al. 2018) and in our model.

Prices. Let us first focus on the change in prices. Figure 9 plots the evolution of the wealth-to-output, Tobin's Q, and returns during the transition. First, the wealth-to-output increases by 10 p.p. compared to the initial steady state. Due to imperfect competition, most of the increase in wealth comes from a price effect instead of a quantity effect, and we thus capture the qualitative evolution of wealth in the U.S. Secondly, the Tobin's Q moves from 0.84 in 1970 to 0.9 in 2020, due to those valuation effects. Finally, the illiquidity premium increases as the return on the safe asset decreases while the return on the illiquid assets increases slightly due to the unexpected nature of capital gains. The path followed by returns is consistent with the main finding of Reis (2022), who empirically observed a relatively stable return on capital over the last 20 years, combined with a fall in the return on the liquid asset.

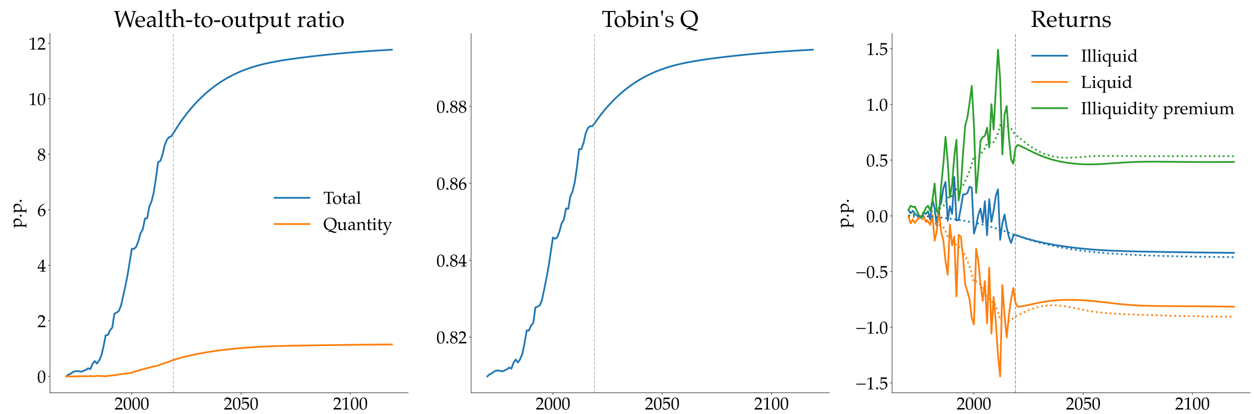

Figure 9: Evolution of prices in the model

Note: The figure on the left shows the p.p. increase in the wealth-to-output ratio in blue and the increase in the physical capital stock to output in orange. The middle graph shows the evolution of the Tobin's Q. The right figure plots the evolution of the returns, with the dotted line showing the evolution of returns in the perfect competition case. Note that the changes are volatile until 2020 because we use the actual changes in permanent labor income observed in the data. They are smooth afterward because of the lack of additional shocks.

All those trends continue even after permanent labor income inequality stops increasing. Specifically, our model then predicts a long-term decrease in the return on capital that will remain modest compared to the decrease in the safe asset, increasing the illiquidity premium. This happens for two reasons. First, while capital gains push up the short-term return on the illiquid assets, the long-term return on the illiquid asset is determined by the marginal productivity of capital. As we have shown before, with imperfect competition, most of the increase in wealth comes from a valuation effect, and the economy accumulates very little physical capital. The long-term marginal productivity of capital will thus decrease very little, and the same will be true of the return on the illiquid asset. Secondly, the elasticity of the demand for safe assets from the government with respect to the interest rate r^b is zero, while the elasticity of the demand for risky assets of the firms with respect to the interest rate r^a is negative. Hence, when the return on the risky market falls, the demand from firms increases, which limits the fall in the return on the risky market. On the safe market, the demand from the government does not increase with the fall in return on the safe market. Therefore, the return on the safe market falls by more than the return on the risky market.[20]

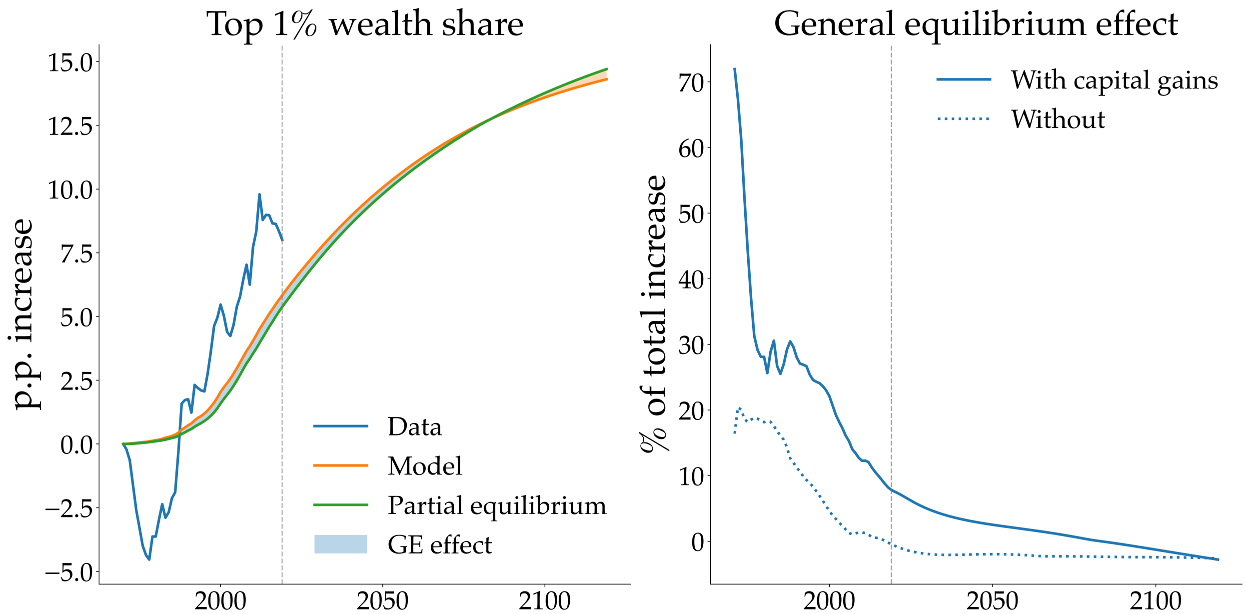

Figure 10: Increase in the top 1% wealth share and general equilibrium effect

Note: The figure on the left shows the p.p. increase in the top 1% wealth share in the model and the data. The orange line shows the increase in general equilibrium, and the green line shows the increase in partial equilibrium, keeping returns and wages constant. The shaded area between the two is the general equilibrium effect. The figure on the right displays this general equilibrium effect as a percentage of the total increase in wealth inequality in the model. The dotted blue line shows the same general equilibrium effect in a model with perfect competition and no capital gains.

Wealth inequality. We can now focus on the response of wealth inequality, which we decompose in a partial equilibrium and a general equilibrium effect. To isolate the general equilibrium effect, we run a transition in partial equilibrium, keeping prices constant and only allowing the parameters $\{z_s\}$ to change. We then run a transition in general equilibrium, adjusting all prices to maintain market clearings. We then define the general equilibrium effect on the top wealth share as

$$GE_t = \frac{\text{Top wealth share}_t^{\text{general}} - \text{Top wealth share}_t^{\text{partial}}}{\text{Top wealth share}_t^{\text{general}} - \text{Top wealth share}_{ss}^{\text{general}}}.$$

Figure 10 displays the main result of our model. The left panel shows the increase in the top 1% wealth share due to the increase in permanent labor income, while the right panel

[20]This is driven by the fact that our model's supply of safe assets consists exclusively of public debt and that we assume that public debt remains constant in the transition. This is obviously a simplifying assumption but is in line with the fact that the share of risky assets in households' portfolios has increased significantly over the past 50 years.

shows our measure of the general equilibrium effect.

As shown in the left panel, our model matches both the speed and the magnitude of the increase in the top 1% wealth share and predicts that this trend will continue in the future. Most of this increase is driven by a partial equilibrium effect, represented by the green line in the left panel of Figure 10. High-productivity households receive higher wages and can thus accumulate more wealth. This effect is amplified by the fact that their marginal propensity to save increases as they become richer. At the bottom of the labor income distribution, the opposite effect occurs: poorer households dissave following a shock on their permanent level of labor income, and their marginal propensity to save decreases as well.

However, as we mentioned before, because of the non-homothetic nature of the taste for wealth, the increase in wealth from the high-productivity households is larger than the dissaving from the poorer households, which increases aggregate savings and changes the equilibrium prices. The effect of this price change on the top 1% wealth share is displayed by the blue-shaded area in the left panel of Figure 11 and by the blue line in the right panel. In 2020, the general equilibrium effect contributed 10% of this increase in the top 1% wealth share and stayed positive until approximately 2100. This means that the change in prices due to a change in permanent labor income inequality increases the speed at which the model's top 1% wealth share increases. It becomes negative after that point, meaning that, over the very long term, the price change actually decreases wealth inequality, as the lower returns negatively impact the wealthy.

Decomposition of the GE effect. We can further decompose this general equilibrium effect between a return and wage effect. Figure 11 shows the result of this decomposition. First, the change in returns increases wealth inequality initially, as richer households, who own mostly illiquid assets, enjoy the capital gains, while poorer households, who own mostly liquid wealth, dissave more as the returns they face decrease quickly. Over the long term, however, both returns decline, and this return effect becomes negative.

Secondly, the increase in the stock of physical capital also increases wages, creating a "trickle-down" effect that dampens the top 1% wealth share slightly over the long term. This effect is negative, as it mostly benefits the poorest households, who get most of their income from labor. However, because the stock of capital increases very little, this effect

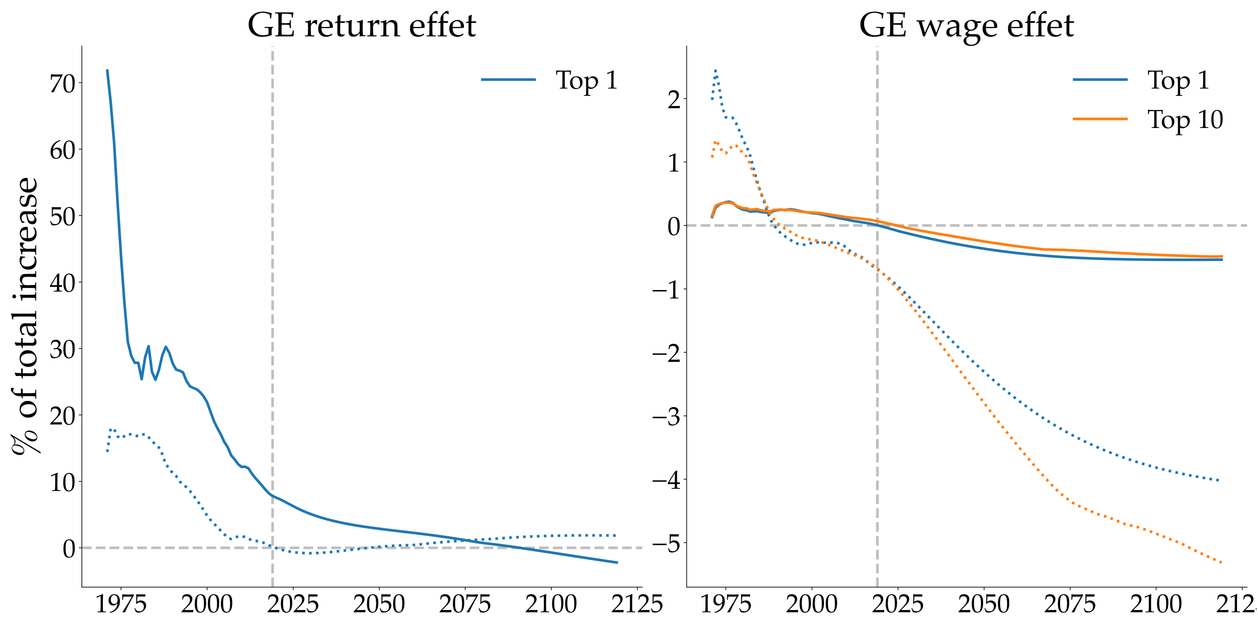

Figure 11: Decomposition of the general equilibrium effect on the top wealth shares

Note: This figure decomposes the general equilibrium effect between a wage effect and a return effect.

is quantitatively very small and does not significantly decrease the top 1% wealth share. In contrast, the dotted line shows the effect of this channel in a model with perfect competition. In this model, there is a one-to-one increase in aggregate savings and the stock of physical capital: wages increase by more, and this channel decreases the top 1% wealth share by 5% in the long run.

5 Conclusion

In this paper, we have studied the specific role of capital gains to account for the rise in wealth inequality in the US since 1970. Our contribution is threefold. (1) In a simple analytical model, we identify the preferences consistent with micro evidence on marginal propensity to save out of a permanent income shock. (2) In a quantitative model calibrated on US data, we endogenously generate valuation effects that create an excess return on the equity market. (3) We quantify the size of this capital gain channel by wealth groups and over time.

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A Analytical model

A.1 Proposition 1

The Lagrangian of our maximization problem is the following:

$$\mathcal{L}(\{s_{i,t+1}\},\{\lambda_{i,t}\}) = \sum_{t=0}^{\infty} \beta_i^t [u(z_i + (1+q_t)s_{i,t} - q_t s_{i,t+1}) + \lambda_{i,t} s_{i,t+1}]$$

Taking the FOC with respect to $s_{i,t+1}$, we get:

$$q_t u'(c_{i,t}) = \beta_i (1+q_{t+1}) u'(c_{i,t+1}) + \lambda_{i,t}$$

We will ignore the borrowing constraint and check ex-post that the sequence $\{a_{t+1}^i\}$ does indeed satisfy the borrowing constraint.

The Euler equation of the household problem gives us $c_{i,t}$ as a function of $c_{i,t-1}$:

$$\forall t,\, c_{i,t} = \left(\beta \frac{1+q_{t+1}}{q_t}\right)^{\frac{1}{\sigma}} c_{i,t-1}$$

As it holds for the two agents, taking the ratio of the two Euler equations, we get that:

$$\forall t,\, \frac{c_{1,t}}{c_{1,t-1}} = \frac{c_{2,t}}{c_{2,t-1}}$$

Since we have that $c_{1,t-1} + c_{2,t-1} = c_{1,t} + c_{2,t} = 2, \quad \forall t$, consumption is necessarily constant over time. This is only possible if:

$$\forall t,\, \beta \frac{1+q_{t+1}}{q_t} = 1$$

As consumption is constant over time, the asset position remains the same, and we have that:

$$s_{i,t+1} = s_{i,0} \forall i \in \{1,2\}, \forall t \geq 0.$$

Hence, the borrowing constraint is indeed satisfied at all times.

A.2 Proposition 2

At the steady state, consumption is constant and the equilibrium allocation $\{r, s_1, s_2, \lambda_1, \lambda_2\}$ is characterized by the following four equations:

$$1 = \beta_1 \left(1 + \frac{1}{q}\right) + \lambda_1 / u'(1 - z + 1 - s),$$

$$1 = \beta_2 \left(1 + \frac{1}{q}\right) + \lambda_2 / u'(z + s),$$

$$\lambda_1 (1 - z + 1 - s) = 0,$$

$$\lambda_2 (z + s) = 0.$$

We are going to prove proposition 2.2 by contradiction:

First case: assuming that the borrowing constraint does not bind for the two agents. It must be the case that $\lambda_1 = \lambda_2 = 0$. Equations (3) and (4) imply that :

$$1 + \frac{1}{q} = \frac{1}{\beta_1} = \frac{1}{\beta_2}$$

This is not possible since we assumed that $\beta_1 \neq \beta_2$.

Second case: the non-negativity constraint holds for agent 2 and not for agent 1. In that case, $\lambda_2 > 0$ and $\lambda_1 = 0$. Putting equations (3) and (4) together, we have that:

$$\beta_1 \left(1 + \frac{1}{q}\right) = \beta_2 \left(1 + \frac{1}{q}\right) + \lambda_2 / u'(c_2)$$

This is not possible since $\lambda_2 / u'(c_2) > 0$ as marginal utility is always positive and we assumed that $\beta_2 > \beta_1$.

Third case: the borrowing constraint holds for the two agents. This is trivially impossible, as the market clearing condition on the equity market would not hold.

The only possibility is that $s_1 = 0$, and $1 + \frac{1}{q} = \frac{1}{\beta_2}$ $\square$

A.3 Proposition 3

When we include a taste for wealth $\gamma > 0$, the Lagrangian of our maximization problem is the following:

$$\mathcal{L}(\{s_{i,t+1}\}, \{\lambda_t^i\}) = \sum_{t=0}^{\infty} \beta^t [u(z_i + (1+q_t)s_{i,t} - s_{i,t+1}) + \gamma v(q_t s_{i,t}) + \lambda_{i,t} s_{i,t+1}].$$

Taking the FOC with respect to $s_{i,t+1}$, we get:

$$q_t u'(c_{i,t}) = \beta[(1+q_{t+1})u'(c_{i,t+1}) + \gamma v'(a_{i,t+1}) + \lambda_{i,t}].$$

At the steady state for agent 1:

$$u'(1-z+1-s) = \beta(1+\frac{1}{q})u'(1-z+1-s) + \beta\gamma v'(q(1-s)) + \lambda_1,$$

$$\iff \frac{1}{\beta} = 1 + \frac{1}{q} + \gamma \frac{v'(q(1-s))}{u'(1-z+1-s)} + \frac{\lambda_1}{u'(1-z+1-s)}.$$

For Agent 2:

$$u'(z+s) = \beta(1+\frac{1}{q})u'(z+s) + \beta\gamma v'(qs) + \lambda_2,$$

$$\iff \frac{1}{\beta} = 1 + \frac{1}{q} + \gamma \frac{v'(qs)}{u'(z+s)} + \frac{\lambda_2}{u'(z+s)}.$$

Our steady-state allocation $\{s, q, \lambda_1, \lambda_2\}$ is characterized by the following system of four equations:

$$\begin{cases} \frac{1}{\beta} = 1 + \frac{1}{q} + \gamma \frac{v'(q(1-s))}{u'(1-z+1-s)} + \frac{\lambda_1}{u'(1-z+1-s)}, \\ \frac{1}{\beta} = 1 + r + \gamma \frac{v'(qs)}{u'(z+s)} + \frac{\lambda_2}{u'(z+s)}, \\ \lambda_1 s = 0, \\ \lambda_2(1-s) = 0. \end{cases}$$

When there is an interior solution, putting the first two equations together gives the equilibrium level of equity holdings s as a function of q :

$$\frac{v'(q(1-s))}{u'(1-z+1-s)} = \frac{v'(qs)}{u'(z+s)}.$$

A.4 Proposition 4

When $\sigma = \Sigma$ and assuming that the non-negativity constraint on equity holdings, the equation equating the two marginal rates of substitution becomes:

$$
\begin{aligned}
&\frac{qs+\zeta}{s+z} = \frac{q(1-s)+\zeta}{1-s+1-z} \\
&(qs+\zeta)(1-s+1-z) = (q(1-s)+\zeta)(s+z) \\
&2(qs+\zeta) - s(qs+\zeta) - z(qs+\zeta) = s(q(1-s)+\zeta) + z(q(1-s)+\zeta) \\
&2qs + 2\zeta - qs^2 - \zeta s - zqs - z\zeta = qs - qs^2 + \zeta s + zq - zqs + z\zeta \\
&(q-2\zeta)s + 2\zeta = zq + 2z\zeta \\
&s = \frac{zq + 2z\zeta - 2\zeta}{q - 2\zeta}
\end{aligned}
$$

In the general case (incorporating binding constraints), equity holdings are given by:

$$s = \min\left\{\frac{zq + 2z\zeta - 2\zeta}{q - 2\zeta}, 1\right\}$$

A.5 Proposition 5

In the case of log utility $\sigma = \Sigma = 1$ and $\zeta = 0$, equity holding becomes:

$$s = z$$

Plugging that in the Euler equation of one of the two agents, we recover the price of the equity share:

$$1 + \frac{1}{q} = \frac{1}{\beta} - \gamma\frac{s+z}{sq}$$

Plugging the solution for s:

$$1 + \frac{1}{q} = \frac{1}{\beta} - \gamma\frac{2}{q} \iff \frac{1+2\gamma}{q} = \frac{1}{\beta} - 1 \iff q = \frac{1+2\gamma}{\frac{1}{\beta} - 1}$$

$$\frac{\partial q}{\partial \beta} > 0, \frac{\partial q}{\partial \gamma} > 0, \text{ and } \frac{\partial q}{\partial z} = 0$$

The price of the Lucas tree is not a function of labor income inequality.

A.6 Proposition 6

Assuming an interior solution:

$$s = \frac{zq + 2y\zeta - 2\zeta}{q - 2\zeta}$$

Plugging that in the Euler equation of one agent:

$$\begin{aligned}
1 + \frac{1}{q} &= \frac{1}{\beta} - \gamma \frac{\frac{zq+2z\zeta-2\zeta}{q-2\zeta} + z}{q\frac{zq+2z\zeta-2\zeta}{q-2\zeta} + \zeta} \\
1 + \frac{1}{q} &= \frac{1}{\beta} - \gamma \frac{zq + 2z\zeta - 2\zeta + z(q - 2\zeta)}{zq^2 + 2z\zeta q - 2\zeta q + \zeta(q - 2\zeta)} \\
1 + \frac{1}{q} &= \frac{1}{\beta} - \gamma \frac{2zq - 2\zeta}{zq^2 + 2z\zeta q - \zeta q - 2\zeta^2} \\
(\frac{1}{\beta} - 1)&(zq^2 + 2z\zeta q - \zeta q - 2\zeta^2) - \gamma(2zq - 2\zeta) - \frac{1}{q}(zq^2 + 2z\zeta q - \zeta q - 2\zeta^2) = 0 \\
(\frac{1}{\beta} - 1)&zq^2 + (\frac{1}{\beta} - 1)(2z\zeta - \zeta)q - 2(\frac{1}{\beta} - 1)\zeta^2 - 2\gamma zq + 2\gamma\zeta - zq - 2z\zeta + \zeta + 2\frac{\zeta^2}{q} = 0 \\
(\frac{1}{\beta} - 1)&zq^3 + \left[(\frac{1}{\beta} - 1)\zeta(2z - 1) - 2\gamma z - z\right]q^2 + \left[2\gamma - 2z + 1 - 2(\frac{1}{\beta} - 1)\zeta\right]\zeta q + 2\zeta^2 = 0
\end{aligned}$$

Notice that this third-order polynomial has a root equal to $\frac{\zeta}{y}$. This root is not a solution to our problem as it is inconsistent with the non-negativity constraint on equity holding. To see that:

$$s = \frac{\zeta + 2z\zeta - 2\zeta}{\frac{\zeta}{z} - 2\zeta} = \frac{2z - 1}{\frac{1}{z} - 2} < 0$$

Hence, we can rewrite the last equation as being equal to:

$$(q - \frac{\zeta}{y})((1 - \beta)q^2 + [2\zeta - (2\beta\gamma + 2\beta\zeta + \beta)]q - 2\beta\zeta) = 0$$

And we can ignore the first term and only solve the following second-order polynomial:

$$(1 - \beta)q^2 + [2\zeta - (2\beta\gamma + 2\beta\zeta + \beta)]q - 2\beta\zeta = 0$$

Computing the discriminant:

$$\Delta = [2\zeta - (2\beta\gamma + 2\beta\zeta + \beta)]^2 + 8(1-\beta)\beta\zeta > 0$$

The second-order polynomial takes two roots:

$$q_1 = \frac{-b - \sqrt{\Delta}}{2a} \text{ and } q_2 = \frac{-b + \sqrt{\Delta}}{2a}$$

The root q_1 is strictly lower than 0. Indeed: $(1-\beta)\beta\zeta > 0$, and so

$\Delta > [2\zeta - (2\beta\gamma + 2\beta\zeta + \beta)]^2$.

Hence:

$$q_1 = \frac{-b - \sqrt{\Delta}}{2a} < \frac{-[2\zeta - (2\beta\gamma + 2\beta\zeta + \beta)] - \sqrt{[2\zeta - (2\beta\gamma + 2\beta\zeta + \beta)]^2}}{2(1-\beta)} = 0$$

Hence, we do have that $q_1 < 0$ and we know that q_1 is not a solution to our problem.

The equilibrium solution price is hence given by:

$$q^* = \frac{\beta\gamma + \beta\zeta + \frac{\beta}{2} - \zeta + \frac{\sqrt{4\beta^2\gamma^2 + 8\beta^2\gamma\zeta + 4\beta^2\gamma + 4\beta^2\zeta^2 - 4\beta^2\zeta + \beta^2 - 8\beta\gamma\zeta - 8\beta\zeta^2 + 4\beta\zeta + 4\zeta^2}}{2}}{1 - \frac{1}{\beta}}$$

Crucially, $dq^*/dy = 0$ and the labor income distribution does not affect the equilibrium price of equity.

A.7 Proposition 2.7

Taking the Euler equation of agent 2:

$$\frac{1}{\beta} - 1 - \frac{1}{q} = \gamma \frac{(z+s)^\sigma}{(qs+\zeta)^\Sigma}$$

And differentiating with respect to z:

$$\frac{\frac{dq}{dz}}{q^2} = \gamma \frac{\sigma(1 + \frac{ds}{dz})(z+s)^{\sigma-1}(qs+\zeta)^\Sigma - \Sigma(s\frac{dq}{dz} + q\frac{ds}{dz})(qs+\zeta)^{\Sigma-1}(z+s)^\sigma}{(qs+\zeta)^{2\Sigma}}$$

$$\iff \frac{\frac{dq}{dz}}{q^2} = \gamma \frac{\sigma(1+\frac{ds}{dz})(qs+\zeta) - \Sigma(s\frac{dq}{dz} + q\frac{ds}{dz})(z+s)}{(qs+\zeta)^{2\Sigma}}(z+s)^{\sigma-1}(qs+\zeta)^{\Sigma-1}$$

$$\iff \frac{\frac{dq}{dz}}{q^2} = \gamma \frac{(z+s)^{\sigma-1}(qs+\zeta)^{\Sigma-1}}{(qs+\zeta)^{2\Sigma}}[\sigma(qs+\zeta) + \sigma(qs+\zeta)\frac{ds}{dz} - \Sigma(z+s)s\frac{dq}{dz} - \Sigma(z+s)q\frac{ds}{dz}]$$

$$\iff \frac{\frac{dq}{dz}}{q^2} = \gamma \frac{(z+s)^{\sigma-1}(qs+\zeta)^{\Sigma-1}}{(qs+\zeta)^{2\Sigma}}[\sigma(qs+\zeta) + [\sigma(qs+\zeta) - \Sigma(z+s)q]\frac{ds}{dz} - \Sigma(z+s)s\frac{dq}{dz}]$$

$$\iff \frac{(qs+\zeta)^{2\Sigma}}{\gamma q^2 (z+s)^{\sigma-1}(qs+\zeta)^{\Sigma-1}}\frac{dq}{dz} = \sigma(qs+\zeta) + [\sigma(qs+\zeta) - \Sigma(z+s)q]\frac{ds}{dz} - \Sigma(z+s)s\frac{dq}{dz}$$

$$\iff 0 = \sigma(qs+\zeta) + [\sigma(qs+\zeta) - \Sigma(z+s)q]\frac{ds}{dz} - \left[\Sigma(z+s)s + \frac{(qs+\zeta)^{2\Sigma}}{\gamma q^2 (z+s)^{\sigma-1}(qs+\zeta)^{\Sigma-1}}\right]\frac{dq}{dz}$$

$$\iff 0 = a_1 + b_1\frac{ds}{dz} - c_1\frac{dq}{dz}$$

With $a_1 \equiv \sigma(qs+\zeta)$, $b_1 \equiv \sigma(qs+\zeta) - \Sigma(z+s)q$, $c_1 \equiv \Sigma(z+s)s + \dfrac{(qs+\zeta)^{\Sigma+1}}{\gamma q^2 (z+s)^{\sigma-1}}$

We need a second equation as we no longer have a closed-form solution for s. Taking the Euler equation of agent 1:

$$\frac{1}{\beta} - 1 - \frac{1}{q} = \gamma \frac{(2-z-s)^{\sigma}}{(q(1-s)+\zeta)^{\Sigma}}$$

And differentiating with respect to z:

$$\frac{\frac{dq}{dz}}{q^2} = \gamma \frac{\sigma(-1-\frac{ds}{dz})(2-z-s)^{\sigma-1}(q(1-s)+\zeta)^{\Sigma} - \Sigma((1-s)\frac{dq}{dz} - q\frac{ds}{dz})(q(1-s)+\zeta)^{\Sigma-1}(2-z-s)^{\sigma}}{(q(1-s)+\zeta)^{2\Sigma}}$$

$$\iff \frac{\frac{dq}{dz}}{q^2} = \gamma \frac{\sigma(-1-\frac{ds}{dz})(q(1-s)+\zeta) - \Sigma((1-s)\frac{dq}{dz} - q\frac{ds}{dz})(2-z-s)}{(q(1-s)+\zeta)^{2\Sigma}}(2-z-s)^{\sigma-1}(q(1-s)+\zeta)^{\Sigma-1}$$

$$\iff \frac{\frac{dq}{dz}}{q^2} = \gamma \frac{(2-z-s)^{\sigma-1}(q(1-s)+\zeta)^{\Sigma-1}}{(q(1-s)+\zeta)^{2\Sigma}}[\sigma(-1-\frac{ds}{dz})(q(1-s)+\zeta) - \Sigma((1-s)\frac{dq}{dz} - q\frac{ds}{dz})(2-z-s)]$$

$$\iff \frac{\frac{dq}{dz}}{q^2} = \gamma \frac{(2-z-s)^{\sigma-1}(q(1-s)+\zeta)^{\Sigma-1}}{(q(1-s)+\zeta)^{2\Sigma}}[-\sigma(q(1-s)+\zeta) - \frac{ds}{dz}\sigma(q(1-s)+\zeta) - \Sigma(1-s)\frac{dq}{dz}(2-z-s) + \Sigma q\frac{ds}{dz}(2-z-s)]$$

$$\iff \frac{\frac{dq}{dz}}{q^2} = \gamma \frac{(2-z-s)^{\sigma-1}(q(1-s)+\zeta)^{\Sigma-1}}{(q(1-s)+\zeta)^{2\Sigma}}[-\sigma(q(1-s)+\zeta) + [\Sigma q(2-z-s) - \sigma(q(1-s)+\zeta)]\frac{ds}{dz} - \Sigma(1-s)\frac{dq}{dz}(2-z-s)]$$

$$\iff \frac{(q(1-s)+\zeta)^{2\Sigma}}{\gamma q^2 (2-z-s)^{\sigma-1}(q(1-s)+\zeta)^{\Sigma-1}}\frac{dq}{dz} = -\sigma(q(1-s)+\zeta) + [\Sigma q(2-z-s) - \sigma(q(1-s)+\zeta)]\frac{ds}{dz} - \Sigma(1-s)(2-z-s)\frac{dq}{dz}$$

$$\iff 0 = -\sigma(q(1-s)+\zeta) + [\Sigma q(2-z-s) - \sigma(q(1-s)+\zeta)]\frac{ds}{dz} - \left[\Sigma(1-s)(2-z-s) + \frac{(q(1-s)+\zeta)^{2\Sigma}}{\gamma q^2 (2-z-s)^{\sigma-1}(q(1-s)+\zeta)^{\Sigma-1}}\right]\frac{dq}{dz}$$

$$\iff 0 = a_2 + b_2\frac{ds}{dz} - c_2\frac{dq}{dz}$$

With $a_2 \equiv -\sigma(q(1-s)+\zeta)$, $b_2 \equiv \Sigma q(2-z-s) - \sigma(q(1-s)+\zeta)$, and $c_2 \equiv \Sigma(1-s)(2-z-s) + \frac{(q(1-s)+\zeta)^{\Sigma+1}}{\gamma q^2(2-z-s)^{\sigma-1}}$

The last expression gives us $\frac{ds}{dz}$ as a function of $\frac{dq}{dz}$:

$$\iff \frac{ds}{dz} = \frac{c_2\frac{dq}{dz} - a_2}{b_2}$$

Plugging that in the first equation:

$$0 = a_1 + b_1\frac{c_2\frac{dq}{dz} - a_2}{b_2} - c_1\frac{dq}{dz}$$

$$\iff 0 = a_1 - \frac{a_2 b_1}{b_2} + \frac{b_1 c_2}{b_2}\frac{dq}{dz} - c_1\frac{dq}{dz}$$

$$\iff 0 = a_1 - \frac{a_2 b_1}{b_2} + \left[\frac{b_1 c_2}{b_2} - c_1\right]\frac{dq}{dz}$$

$$\frac{dq}{dz} = \frac{a_1 - \frac{a_2 b_1}{b_2}}{\frac{b_1 c_2}{b_2} - c_1} = \frac{a_1 b_2 - a_2 b_1}{b_2 c_1 - b_1 c_2}$$

Or:

$$\frac{dq}{dz} = \frac{\sigma(qs+\zeta)(\Sigma q(2-z-s) - \sigma(q(1-s)+\zeta)) + \sigma(q(1-s)+\zeta)(\sigma(qs+\zeta) - \Sigma(z+}{(\Sigma q(2-z-s) - \sigma(q(1-s)+\zeta))(\Sigma(z+s)s + \frac{(qs+\zeta)^{\Sigma+1}}{\gamma q^2(z+s)^{\sigma-1}}) - (\sigma(qs+\zeta) - \Sigma(z+s)q)(\Sigma(1-s)(2-}$$

Evaluating this expression around the $\sigma = \Sigma = 1$ case:

$$a_1 b_2 = \sigma(qs+\zeta)(\Sigma q(2-z-s) - \sigma(q(1-s)+\zeta))$$

$$\iff a_1 b_2 = (qs+\zeta)(2q - zq - qs - q(1-s) - \zeta)$$

$$\iff a_1 b_2 = (qs+\zeta)(2q - zq - qs - q + qs - \zeta)$$

$$\iff a_1 b_2 = (qs+\zeta)(q - zq - \zeta)$$

$$a_2 b_1 = -\sigma(q(1-s)+\zeta)(\sigma(qs+\zeta) + \Sigma(z+s)q)$$

$$\iff a_2 b_1 = -(q(1-s)+\zeta)(qs+\zeta+zq+qs)$$

$$\iff a_2 b_1 = -(q(1-s)+\zeta)(2qs+\zeta+zq)$$

Computing the numerator:

$$a_1 b_2 - a_2 b_1 = (qs+\zeta)(q(1-z)-\zeta) + (q(1-s)+\zeta)(2qs+\zeta+zq)$$

$$\iff a_1 b_2 - a_2 b_1 = (qs+\zeta)q(1-z) - \zeta(qs+\zeta) + q(1-s)(2qs+\zeta+zq) + \zeta(2qs+\zeta+zq)$$
$$\iff a_1 b_2 - a_2 b_1 = (qs+\zeta)q(1-z) + q(1-s)(2qs+\zeta+zq) + \zeta(qs+zq) > 0$$

Now, moving to the denominator:

$$b_1 c_2 = [\sigma(qs+\zeta) + \Sigma(z+s)q][\Sigma(1-s)(2-z-s) + \frac{(q(1-s)+\zeta)^{\Sigma+1}}{\gamma q^2 (2-z-s)^{\sigma-1}}]$$

$$\iff b_1 c_2 = [(qs+\zeta) + (z+s)q][(1-s)(2-z-s) + \frac{(q(1-s)+\zeta)^2}{\gamma q^2}]$$

$$\iff b_1 c_2 = [2qs+\zeta+zq][(1-s)(2-z-s) + \frac{(q(1-s)+\zeta)^2}{\gamma q^2}]$$

$$b_2 c_1 = [\Sigma q(2-z-s) - \sigma(q(1-s)+\zeta)][\Sigma(z+s)s + \frac{(qs+\zeta)^{\Sigma+1}}{\gamma q^2 (z+s)^{\sigma-1}}]$$

$$\iff b_2 c_1 = [q(2-z-s) - (q(1-s)+\zeta)][(z+s)s + \frac{(qs+\zeta)^2}{\gamma q^2}]$$

$$\iff b_2 c_1 = [2q - zq - qs - q + qs - \zeta][(z+s)s + \frac{(qs+\zeta)^2}{\gamma q^2}]$$

$$\iff b_2 c_1 = [q - zq - \zeta][(z+s)s + \frac{(qs+\zeta)^2}{\gamma q^2}]$$

$$\iff b_2 c_1 = [q(1-z) - \zeta][(z+s)s + \frac{(qs+\zeta)^2}{\gamma q^2}]$$

Computing the denominator:

$$b_1 c_2 - b_2 c_1 = [2qs+\zeta+zq][(1-s)(2-z-s) + \frac{(q(1-s)+\zeta)^2}{\gamma q^2}] - [q(1-z)-\zeta][(z+s)s + \frac{(qs+\zeta)^2}{\gamma q^2}]$$

Left to show that it is positive which implies that $\frac{dq}{dz} > 0$.

Computing the denominator:

$$(\Sigma q(2-z-s) - \sigma(q(1-s)+\zeta))(\Sigma(z+s)s + \frac{(qs+\zeta)^{\Sigma+1}}{\gamma q^2 (z+s)^{\sigma-1}}) - (\sigma(qs+\zeta) - \Sigma(z+s)q)(\Sigma(1-s)(2-z-s$$

Using the fact that:

$$\frac{(z+s)^{\sigma}}{(qs+\zeta)^{\Sigma}} = \frac{1}{\gamma}[\frac{1}{\beta}-1-\frac{1}{q}] \iff \frac{(qs+\zeta)^{\Sigma}}{(z+s)^{\sigma}} = \gamma[\frac{1}{\beta}-1-\frac{1}{q}]^{-1}$$

And :

$$\frac{(2-z-s)^{\sigma}}{(q(1-s)+\zeta)^{\Sigma}} = \frac{1}{\gamma}[\frac{1}{\beta}-1-\frac{1}{q}] \iff \frac{(q(1-s)+\zeta)^{\Sigma}}{(2-z-s)^{\sigma}} = \gamma[\frac{1}{\beta}-1-\frac{1}{q}]^{-1}$$

Plugging that in the denominator:

$$(\Sigma q(2-z-s)-\sigma(q(1-s)+\zeta))(\Sigma(z+s)s+\gamma[\frac{1}{\beta}-1-\frac{1}{q}]^{-1}(qs+\zeta)(z+s))-(\sigma(qs+\zeta)-\Sigma(z+s)q)(\Sigma($$

$$=(z+s)(\Sigma q(2-z-s)-\sigma(q(1-s)+\zeta))(\Sigma s+\gamma[\frac{1}{\beta}-1-\frac{1}{q}]^{-1}(qs+\zeta))-(2-z-s)(\sigma(qs+\zeta)-\Sigma(z+s$$

B Heterogeneous-agent model in the analytical section

This section describes the one-asset and two-asset models in Section 2.

B.1 Households

The problem of the household with one-asset writes

$$V(a_t,e_t,z) = \max_{a_{t+1}}\{u(c_t)+v(a_t)+\beta\mathbb{E}_e V(a_{t+1},e_{t+1},z)\};$$
$$\text{subject to } c_t+a_{t+1} = a_t(1+r_t)+ze_tw_t;$$
$$a_{t+1}\geq 0.$$

The two-asset version of the problem writes

$$V(a_t,b_t,e_t,z) = \max_{a_{t+1},b_{t+1}}\{u(c_t)+v(a_t+b_t)+\beta\mathbb{E}_s V(a_{t+1},b_{t+1},e_{t+1},z)\}$$
$$\text{subject to } c_t+a_{t+1}+b_{t+1}+\chi(a_{t+1},a_t) = (1+r_t^a)a_t+(1+r_t^b)b_t+e_tzw_t$$
$$a_{t+1}\geq 0, \quad b_{t+1}\geq \overline{b}.$$

In this two-asset version of the model, we assume that capital gains are paid only to the illiquid account. That is, the excess returns at period 0 $\frac{q_1 - q_{ss}}{q_{ss}}$ are paid only to the owners of illiquid assets.

B.1.1 Firms

Deterministic Lucas tree. There is a Lucas tree in unit-net supply that provides one unit of consumption good every period:

$$y_t = s_t = 1.$$

We assume that a share α is paid to the owners of the Lucas tree, so that

$$d_t = \alpha y_t.$$

The return on the Lucas tree is given by

$$r_t = \frac{d_t + q_{t+1}}{q_t} - 1.$$

The rest of the output is paid to households through wages

$$w_t = (1 - \alpha) y_t.$$

Neoclassical model. The supply side of this section is kept intentionally stylized. In the model with a neo-classical production function and without capital gains, output is given by

$$y_t = c k_t^{\alpha}.$$

We assume zero depreciation of capital so that returns and wages are given by

$$r_t = \alpha c k_t^{\alpha - 1},$$

$$w_t = (1 - \alpha) c k_t^{\alpha}.$$

Calibration. We set $\alpha = 1/3$, and calibrate c so that the initial steady state when inequality is low are the same in the economy with a Lucas tree and with a neo-classical production function. We set β in the household problem so that the initial steady state return r is equal to 5%. γ, ζ and Σ are set to the same values as in the quantitative model described in Section 3.

C Computational appendix

This section describes how we solve the quantitative model of Section 3.

C.1 Household maximization

Given parameters and given a path for $\{r_t^a, r_t^b, w_{i,t}, \tau_{l,t}\}$, a household solves the following dynamic programming problem

$$
\begin{aligned}
V(a_t, b_t, e_t, i) = \max_{\tilde{a_{t+1}}, b_{t+1}} & \{u(c_t) + v(a_t + b_t) + (1-\xi)\beta \mathbb{E}_{z,s} V(a_{t+1}, b_{t+1}, z_{t+1}, i)\} \\
\text{subject to } & c_t + a_{t+1} + b_{t+1} + \chi(a_{t+1}, a_t) = (1 + r_t^a) s_t \tilde{a}_t + (1 + r_t^b) b_t + (1 - \tau_{l,t})(z_t w_{i,t})^{1-\theta} \\
& a_{t+1} \geq 0, \quad b_{t+1} \geq \overline{b}
\end{aligned}
$$

where $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ and $v(a_t + b_t) = \frac{(a_t + b_t + \zeta)^{1-\Sigma}}{1-\Sigma}$.

The first-order conditions of the problem are

$$
\begin{aligned}
u'(c_t) &= (1-\xi)\beta \mathbb{E}_{z,s}[V_a(a_{t+1}, b_{t+1}, z_{t+1}, i)] + \lambda_t \\
u'(c_t)(1 + \chi_1') &= (1-\xi)\beta \mathbb{E}_{z,s}[V_b(a_{t+1}, b_{t+1}, z_{t+1}, i)] + \lambda_t
\end{aligned}
$$

The envelope conditions are

$$
\begin{aligned}
V_a(a_t, b_t, z_t, i) &= u'(c_t)(1 + r_t^b) + v'(a_t + b_t) \\
V_b(a_t, b_t, z_t, i) &= u'(c_t)(1 + r_t^a - \chi_2) + v'(a_t + b_t)
\end{aligned}
$$

Note that in Section 3, we defined the idiosyncratic return shock as a shock on r_t^a. For computational purposes, we actually implement a shock on the asset position of the household a_t. In this notation, $\tilde{a}_{t+1}$ denotes the choice of the household while $a_{t+1} = s_{t+1}\tilde{a}_{t+1}$ is the effective asset position after the idiosyncratic shock s_{t+1} is realized. Since s_{t+1} is iid, it does not appear as a state variable in the household's problem: at t, the household cares only about its post-shock asset position a_t, and not about its previous choice $\tilde{a}_t$ or the realization of the shock s_t.

To solve this problem, we discretize the AR(1) process using the Rowerhorst method on a grid of 3 points and use the method of endogenous grid points [@carroll2005method]. See the appendix of [@auclert2021using] for additional details on how to implement the EGM method for a two assets problem. The only difference in our setting is that we need to account for the taste for wealth when updating V_a and V_b and take into account the shock on s when computing the expectation.

C.2 Computation of the steady-state

To solve for the steady state, we implement the following algorithm:

1. Given a guess on (r^a, r^b), we solve the problem of the firm using the first-order conditions on capital and wages:

$$
\begin{aligned}
K &= \left(\frac{\kappa ra + \delta - \tau_c \delta}{\alpha Z(1-\tau_c)} \frac{\epsilon}{\epsilon - 1}\right)^{\frac{1}{\alpha-1}} \\
z_i &= \omega_i (1-\alpha) \frac{\epsilon - 1}{\epsilon} \frac{Y}{l_i} \\
d &= (1-\tau_c)(Y - \delta K - z) \\
q &= \frac{(1-\tau_c)d}{r^a}
\end{aligned}
$$

2. We use the budget constraint of the government to solve for τ_l

3. Using our guesses (V_a, V_b) and (r^a, r^b) and the associated (z_i, τ_l), we solve the problem of the household using the method described previously.

4. Once we have obtained the policy functions $c(a,b,z,i), a'(a,b,z,i), b'(a,b,z,i)$, we use @young2010 lottery method to compute the associated transition matrix and

stationary distribution. Note that we also need to account for the probability of death (i.e. to restart with zero wealth) and for the idiosyncratic return shock.

5. We can then use the distribution μ_t to evaluate the market clearing equations

$$\int a'(a,b,z,i)d\mu_t - q = error_a$$
$$\int b'(a,b,z,i)d\mu_t - B_{gov} = error_a$$

6. If $\max(|error_a, error_b|) > tol$, we update our guesses using Newton's method.

C.3 Transition following a permanent labor income shock

This section describes how to solve the transition between two steady states, after a shock on the permanent labor income parameters $\{\omega_i\}$.

1. Compute the new terminal steady state associated with the new $\{\omega_i\}$ and obtain the derivative of the value function V_a, V_b.

2. Guess a path on $\{r_t^a, r_t^b\}_{t=0}^T$ where T is sufficiently large enough (700 in our calibration).

3. Given the guess on the interest rates, solve for the equilibrium wages, equity prices and taxes using the first-order condition of the firm and the fiscal rule of the government. As described in Section 4, note that we need to update the initial return on the risky asset that we denote r_0^h to account for the jump in the equity value.

4. Solve the value function problem of the household backward using the value function at the new steady state as a terminal condition.

5. Use the obtained policy function and the initial distribution μ_0 to solve forward the path of distributions $\{\mu\}_{t=0}^T$

6. Check the market clearing conditions for all t

$$\int a'_t(a,b,z,i)d\mu_t - q = error_{t,a}$$
$$\int b'_t(a,b,z,i)d\mu_t - B_{gov} = error_{t,a}$$

7. Update the guess on the returns using a pseudo-Newton's algorithm where the Jacobians are evaluated around the terminal steady state, using the Sequence Space Jacobian method of @auclert2021using.

Because we model the transition as a sequence of unexpected shocks, we need to compute 40 transitions, where the initial conditions are given by the first-period previous transition. That is, for each unexpected inequality shock on $\{\omega\}_s$, we compute the associated terminal steady state and the transition from 0 to T using the previous algorithm. The new transition takes as an initial condition the distribution μ_1, the capital stock K_1, government debt $B_{gov,1}$ and the price of the firm q_1 as its initial condition.

C.4 Decomposition of the general equilibrium effect

We define the general equilibrium effect of a shock on permanent labor income inequality on the wealth distribution as

$$GE_t = \frac{\text{Top wealth share}_t^{\text{general}} - \text{Top wealth share}_t^{\text{partial}}}{\text{Top wealth share}_t^{\text{general}} - \text{Top wealth share}_{ss}^{\text{general}}}$$

To measure this effect, we solve a full transition of our main model, adjusting prices to maintain the market clearing, using the algorithm described in the previous section. We then solve the transition by maintaining prices at their initial level but changing wages to take into account the direct effect of the changes in $\{\omega_s\}$.

Denote $\{w_t^{\text{partial}}\}_s$ the wages accounting only for this direct effect. To compute the partial equilibrium distribution:

1. We compute a new terminal steady state where we fix the prices at their steady state level $(r^a_{ss}, r^b_{ss}, w_t^{\text{partial}})$ and obtain the associated value function of the households

$V(a, b, i)$

2. We use those terminal conditions to solve the problem of the household in the transition, keeping the returns constant but allowing for the new path in wages, and using the distribution at the steady state as a the initial distribution. We obtain a path for the policy functions $a'(a, b, i, t)$ and $b'(a, b, i, t)$

3. We use the policy functions to solve forward the distribution of households $\mu_t(a, b, i)$ and compute the associated wealth shares.

C.5 Model without capital gains

In our model without capital gains, we remove the imperfect competition structure and allow households to own and lend capital to the firms directly. The first-order condition of the firm is now

$$r_t^a = \alpha K^{\alpha-1} L^{1-\alpha} - \delta$$

and the market clearing condition for the risky assets is modified to become

$$A_t^s = K_t.$$

In this case, there is no change in the relative value of capital after a permanent labor income shock and we do not need to adjust the period 0 return on risky assets.

We also set $\alpha = \frac{1}{3}$ in this calibration to match the capital and labor share. The following table describes the initial steady state in both models.