

Online Appendix

From Income to Wealth Inequality: Trickle-Down vs. Capital Gains

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This appendix accompanies the paper “From Income to Wealth Inequality: Trickle-Down vs. Capital Gains.”

A Proof of Analytical Model

A.1 Additional details on the analytical model

The economy is composed of two agents who differ in their level of permanent income z_i . We index agents as 1 and 2 (1 being the rich, 2 the poor, so that $z_1 > z_2$). We also impose that $z_1 + z_2 = 1$, so that an increase in inequality is a mean-preserving spread. They have preferences towards consumption and wealth. Their maximization problem writes

$$\max_{c_t, a_t} V(a_{t-1}) = \frac{c_t^{1-\sigma}}{1-\sigma} + \frac{a_t^{1-\Sigma}}{1-\Sigma} + \beta V(a_t)$$

subject to $a_t + c_t = a_{t-1}(1 + r_t^a) + z_i w_t$. The first-order condition with respect to a_t is

$$c_t^{-\sigma} = a_t^{-\Sigma} + \beta V'(a_t).$$

The envelope condition gives $V'(a_{t-1}) = (1 + r_t^a)c_t^{-\sigma}$, so that $V'(a_t) = (1 + r_{t+1}^a)c_{t+1}^{-\sigma}$ and, the Euler equation is

$$c_t^{-\sigma} = a_t^{-\Sigma} + \beta(1 + r_{t+1}^a)c_{t+1}^{-\sigma}.$$

At the steady-state, $a_t = a_{t-1}$ so that $c = ar + wz$ and

$$1 - \beta(1 + r) = \frac{a^{-\Sigma}}{c^{-\sigma}}.$$

A.2 Proof Proposition 1

Let us start from the asset-market clearing condition $A = a_1 + a_2 = q$. $A(z, w(r), r)$ is a function of z, r and $w(r)$, while the value of the firm is a function of the interest rate $q(r)$. Taking the log on both sides and totally differentiating the asset market equation yields

$$\begin{aligned} dr &= -\frac{\partial \log A}{\varepsilon^A + \varepsilon^q} \\ d \log(q/Y) &= \frac{\varepsilon^q - \varepsilon^Y}{\varepsilon^A + \varepsilon^q} \cdot \partial \log A, \end{aligned}$$

To show that $dr < 0$, we show that (i) $\partial \log A$ is positive, (ii) ε^q is positive, (iii) that ε^A is positive. To show that the wealth to output increases, we finally show that $\varepsilon^q > \varepsilon^Y$.

1. Positivity of $\partial \log A$

Now, let us prove that $\partial \log A > 0$. Rewrite the Euler equation of a household as

$$F(a, z, w, r) \equiv (ar + wz)^\sigma a^{-\Sigma} + \beta(1 + r) - 1 = 0.$$

By the Implicit Function Theorem, provided $F_a \neq 0$, we have

$$\frac{\partial a}{\partial z} = -\frac{F_z}{F_a}.$$

Taking the derivative, we obtain

$$\begin{aligned} F_a &= a^{-\Sigma-1} c^{\sigma-1} [(\sigma - \Sigma)ar - \Sigma wz] \\ F_z &= \sigma w a^{-\Sigma} c^{\sigma-1}. \end{aligned}$$

Define $D \equiv \Sigma\omega z - (\sigma - \Sigma)ar$. We then have

$$\frac{\partial a}{\partial z} = \frac{\sigma\omega \cdot a}{D},$$

which is positive if $D > 0$. The second derivative can be written as

$$\frac{\partial^2 a}{\partial z^2} = \sigma\omega \left(\frac{\partial a}{\partial z} D^{-1} - a D^{-2} \frac{\partial D}{\partial z} \right)$$

with $\partial D / \partial z = \Sigma\omega - (\sigma - \Sigma)r \partial a / \partial z$. Thus,

$$\begin{aligned} \frac{\partial^2 a}{\partial z^2} &= \sigma\omega \left(\frac{\partial a}{\partial z} D^{-1} - a D^{-2} \left(\Sigma\omega - (\sigma - \Sigma)r \frac{\partial a}{\partial z} \right) \right) \\ &= \frac{\sigma\omega}{D} \left(\frac{\partial a}{\partial z} \left(1 + \frac{ar(\sigma - \Sigma)}{D} \right) - \frac{a\Sigma\omega}{D} \right) \\ &= \frac{\sigma\omega a}{D^2} \left(\sigma\omega \left(1 + \frac{ar(\sigma - \Sigma)}{D} \right) - \Sigma\omega \right) \\ &= \frac{\sigma\omega^2 a(\sigma - \Sigma)}{D^2} \left(1 + \frac{\sigma ar}{D} \right) > 0. \end{aligned}$$

So $a(z; r, w)$ is increasing and convex in z . Since $A = a(z_1) + a(z_2)$ and $z_1 + z_2 = 1$, convexity implies that a mean-preserving spread ($dz_1 = -dz_2 > 0$) increases aggregate savings: $\partial \log A > 0$.

2. Positivity of ε^q

For convenience, define

$$\varepsilon^K \equiv -\frac{\partial \log K}{\partial r} = \frac{1}{(1 - \alpha)(r + \delta)}.$$

Recall that $q = K + V$ where $V \equiv \Pi/r$ with $\Pi \equiv \alpha \frac{\mu-1}{\mu} Y$ denoting gross monopoly rents. Since $\varepsilon^Y \equiv -\partial \log Y / \partial r = \alpha \varepsilon^K$, differentiating gives

$$-\frac{\partial q}{\partial r} = K\varepsilon^K + \frac{\Pi\varepsilon^Y + V}{r}.$$

Using $\Pi = rV$, we rewrite the second term as

$$\frac{\Pi\varepsilon^Y + V}{r} = V \left(\varepsilon^Y + \frac{1}{r} \right).$$

Now,

$$\varepsilon^Y + \frac{1}{r} = \frac{\alpha r + (1 - \alpha)(r + \delta)}{r(1 - \alpha)(r + \delta)} = \varepsilon^K \cdot \frac{r + (1 - \alpha)\delta}{r},$$

so that

$$-\frac{\partial q}{\partial r} = \varepsilon^K \left(q + V \frac{(1 - \alpha)\delta}{r} \right).$$

Dividing by q ,

$$\varepsilon^q = \varepsilon^K \left(1 + \frac{V(1 - \alpha)\delta}{q r} \right) > 0, \quad (1)$$

since every factor is positive.

3. Positivity of ε^A

Now let us focus on ε^A . The aggregate semi-elasticity of asset demand can be decomposed as a wealth-weighted sum of individual semi-elasticities:

$$\varepsilon^A = \frac{d \log A}{dr} = \frac{1}{A} \sum_i \frac{da_i}{dr} = \frac{1}{A} \sum_i a_i \varepsilon^{a_i}. \quad (2)$$

Each individual semi-elasticity can in turn be decomposed into a direct effect and an indirect effect through the wage:

$$\varepsilon^{a_i} = \frac{d \log a_i(z, r, w(r))}{dr} = \frac{\partial \log a_i}{\partial r} + \frac{\partial \log a_i}{\partial \log w} \cdot \frac{d \log w}{dr} = \varepsilon^{a_i, r} + \varepsilon^{a_i, w} \varepsilon^{w, r}. \quad (3)$$

From the firm's first-order condition $(r + \delta) = \frac{\alpha}{\mu} ZK^{\alpha-1}$, we obtain

$$\varepsilon^{w, r} \equiv \frac{d \log w}{dr} = -\frac{\mu K}{w}. \quad (4)$$

Applying the Implicit Function Theorem to the household's Euler equation yields

$$\frac{\partial a_i}{\partial r} = \frac{\sigma a_i^2 + \beta a_i^{\Sigma+1} c_i^{1-\sigma}}{D_i}, \quad \frac{\partial a_i}{\partial w} = \frac{\sigma z_i a_i}{D_i}, \quad (5)$$

where $D_i \equiv \Sigma w z_i - (\sigma - \Sigma) r a_i$. Converting to (semi-)elasticities,

$$\varepsilon^{a_i, r} = \frac{\partial \log a_i}{\partial r} = \frac{\sigma a_i + \beta a_i^{\Sigma} c_i^{1-\sigma}}{D_i}, \quad \varepsilon^{a_i, w} = \frac{\partial \log a_i}{\partial \log w} = \frac{\sigma w z_i}{D_i}. \quad (6)$$

Substituting into the aggregate decomposition,

$$\varepsilon^A = \frac{1}{A} \sum_i a_i \left(\frac{\sigma a_i + \beta a_i^\Sigma c_i^{1-\sigma}}{D_i} - \frac{\sigma w z_i}{D_i} \cdot \frac{\mu K}{w} \right) = \frac{\sigma}{A} \sum_i \frac{a_i}{D_i} (a_i - \mu K z_i) + \frac{\beta}{A} \sum_i \frac{a_i^{\Sigma+1} c_i^{1-\sigma}}{D_i}. \quad (7)$$

The second sum is manifestly positive since $a_i^{\Sigma+1} c_i^{1-\sigma} / D_i > 0$ for all i under our maintained assumption $D_i > 0$. For the first sum, we show that $\sum_i (a_i / D_i) (a_i - \mu K z_i) > 0$. Using $a_2 - \mu K z_2 = (A - \mu K) - (a_1 - \mu K z_1)$, we write

$$\sum_i \frac{a_i}{D_i} (a_i - \mu K z_i) = \underbrace{(a_1 - \mu K z_1)}_{>0} \underbrace{\left(\frac{a_1}{D_1} - \frac{a_2}{D_2} \right)}_{>0} + \underbrace{\frac{a_2}{D_2}}_{>0} \underbrace{(A - \mu K)}_{\geq 0}, \quad (8)$$

a sum of two non-negative terms, the first strictly positive. We verify each sign in turn.

- (i) $a_1 - \mu K z_1 > 0$. We show that $a_1 / z_1 > a_2 / z_2$. This suffices because cross-multiplying yields $a_1 z_2 > a_2 z_1$, and substituting $z_2 = 1 - z_1$:

$$a_1(1 - z_1) > a_2 z_1 \iff a_1 > (a_1 + a_2) z_1 = A z_1.$$

Moreover, $A \geq \mu K$ since $A = q = K + \alpha \frac{\mu-1}{\mu} \frac{Y}{r}$ and $\alpha Y = \mu(r + \delta)K$ yield $A = K \left(1 + \frac{(\mu-1)(r+\delta)}{r} \right) \geq \mu K$, so $a_1 > A z_1 \geq \mu K z_1$.

We prove $a_1 / z_1 > a_2 / z_2$ by contradiction. Suppose $a_1 / z_1 \leq a_2 / z_2$, i.e. $a_1 z_2 \leq a_2 z_1$. From the budget constraint $c_i = a_i r + w z_i$,

$$c_1 a_2 - c_2 a_1 = w(a_2 z_1 - a_1 z_2) \geq 0,$$

so $c_1 / c_2 \geq a_1 / a_2$. Now, the Euler equation $c_i^\sigma a_i^{-\Sigma} = 1 - \beta(1 + r)$ takes the same value for both agents, so

$$\left(\frac{c_1}{c_2} \right)^\sigma = \left(\frac{a_1}{a_2} \right)^\Sigma \implies \frac{c_1}{c_2} = \left(\frac{a_1}{a_2} \right)^{\Sigma/\sigma}.$$

Since $a_1 > a_2$ (from $\partial a / \partial z > 0$ and $z_1 > z_2$), we have $a_1 / a_2 > 1$. Because $\Sigma / \sigma < 1$ (the non-homotheticity assumption), concavity of $t \mapsto t^{\Sigma/\sigma}$ implies $(a_1 / a_2)^{\Sigma/\sigma} < a_1 / a_2$, so $c_1 / c_2 < a_1 / a_2$. This contradicts $c_1 / c_2 \geq a_1 / a_2$.

- (ii) $a_1 / D_1 > a_2 / D_2$. Cross-multiplying, $a_1 D_2 - a_2 D_1 = \Sigma w (a_1 z_2 - a_2 z_1) > 0$, where the

inequality follows from $a_1/z_1 > a_2/z_2$ established in (i).

(iii) $A \geq \mu K$. This follows from $A = K(1 + (\mu - 1)(r + \delta)/r) \geq \mu K$, with equality under perfect competition ($\mu = 1$).

4. $\varepsilon^q > \varepsilon^Y$

From the production function, $\varepsilon^Y \equiv -d \log(Y)/dr = \alpha \varepsilon^K$. Using our previous result, we have

$$\varepsilon^q - \varepsilon^Y = \varepsilon^K \left(1 + \frac{V}{q} \frac{(1 - \alpha)\delta}{r} \right) - \alpha \varepsilon^K = \varepsilon^K (1 - \alpha) \left(1 + \frac{V\delta}{q r} \right) > 0.$$

A.3 Proof Proposition 2

We show that $\varepsilon^{a_1} > \varepsilon^{a_2}$, which implies that $\varepsilon^{a_1} > \varepsilon^A$.

Preliminary: the assumption of Proposition 1 implies $D_i > 0$ for $i = 1, 2$.

Recall that $D_i \equiv \Sigma w z_i - (\sigma - \Sigma) r a_i$, so that $D_i > 0$ is equivalent to $\Sigma / (\sigma - \Sigma) > a_i r / (w z_i)$, or equivalently $r a_i / c_i < \Sigma / \sigma$. Since $a_1 / z_1 > a_2 / z_2$ (step 3(i) of Proposition 1), we have $a_1 r / (w z_1) > a_2 r / (w z_2)$, and therefore $D_1 > 0$ implies $D_2 > 0$. Assumption 1 states precisely that $D_1 > 0$, so both D_1 and D_2 are positive.

From equation (7), the individual total semi-elasticity reads

$$\varepsilon^{a_i} = \frac{\sigma(a_i - \mu K z_i) + \beta a_i^\Sigma c_i^{1-\sigma}}{D_i}. \quad (9)$$

Since $D_1, D_2 > 0$, the sign of $\varepsilon^{a_1} - \varepsilon^{a_2}$ is determined by the numerator

$$\mathcal{N} \equiv [\sigma(a_1 - \mu K z_1) + \beta a_1^\Sigma c_1^{1-\sigma}] D_2 - [\sigma(a_2 - \mu K z_2) + \beta a_2^\Sigma c_2^{1-\sigma}] D_1.$$

Separating the σ and β terms,

$$\mathcal{N} = \underbrace{\sigma[(a_1 - \mu K z_1) D_2 - (a_2 - \mu K z_2) D_1]}_{\mathcal{N}_1} + \underbrace{\beta[a_1^\Sigma c_1^{1-\sigma} D_2 - a_2^\Sigma c_2^{1-\sigma} D_1]}_{\mathcal{N}_2}.$$

First term. Expanding,

$$(a_1 - \mu K z_1) D_2 - (a_2 - \mu K z_2) D_1 = (a_1 D_2 - a_2 D_1) - \mu K (z_1 D_2 - z_2 D_1).$$

From step 3(ii) of Proposition 1, $a_1D_2 - a_2D_1 = \Sigma w(a_1z_2 - a_2z_1)$. For the second cross-product, substituting $D_i = \Sigma wz_i - (\sigma - \Sigma)a_i r$ gives

$$z_1D_2 - z_2D_1 = (\sigma - \Sigma)r(a_1z_2 - a_2z_1).$$

Combining,

$$\mathcal{N}_1 = \sigma(a_1z_2 - a_2z_1)[\Sigma w - \mu(\sigma - \Sigma)rK].$$

Since $a_1/z_1 > \mu K$ (step 3(i): $a_1 > Az_1 \geq \mu Kz_1$), the condition $D_1 > 0$ implies $\Sigma w > (\sigma - \Sigma)a_1r/z_1 > \mu(\sigma - \Sigma)Kr$. Together with $a_1z_2 - a_2z_1 > 0$, this yields $\mathcal{N}_1 > 0$.

Second term. From the steady-state Euler equation $c_i^\sigma a_i^{-\Sigma} = \lambda$, where $\lambda \equiv 1 - \beta(1 + r) > 0$, we obtain $a_i^\Sigma = c_i^\sigma / \lambda$ and hence $a_i^\Sigma c_i^{1-\sigma} = c_i / \lambda$. Therefore

$$\mathcal{N}_2 = \frac{\beta}{\lambda}(c_1D_2 - c_2D_1).$$

Using the budget constraint $c_i = a_i r + wz_i$ together with the cross-products computed above,

$$\begin{aligned} c_1D_2 - c_2D_1 &= r(a_1D_2 - a_2D_1) + w(z_1D_2 - z_2D_1) \\ &= r\Sigma w(a_1z_2 - a_2z_1) + w(\sigma - \Sigma)r(a_1z_2 - a_2z_1) \\ &= \sigma wr(a_1z_2 - a_2z_1) > 0, \end{aligned}$$

so $\mathcal{N}_2 > 0$.

Since $\mathcal{N} = \mathcal{N}_1 + \mathcal{N}_2 > 0$, we conclude $\varepsilon^{a_1} > \varepsilon^{a_2}$. From the weighted-average decomposition $\varepsilon^A = (a_1/A)\varepsilon^{a_1} + (a_2/A)\varepsilon^{a_2}$, this implies $\varepsilon^{a_1} > \varepsilon^A$, i.e. $\varepsilon^{a_1} - \varepsilon^A > 0$. Since $dr < 0$, the trickle-down effect $(\varepsilon^{a_1} - \varepsilon^A)dr < 0$: the wealth of the rich declines proportionally more than aggregate wealth.

A.4 Proof of Proposition 3

We maintain $\beta = 0$ throughout.

Welfare derivative. With $\beta = 0$, per-period utility at the steady state is $U_i = c_i^{1-\sigma}/(1 - \sigma) + a_i^{1-\Sigma}/(1 - \Sigma)$. By the envelope theorem, the welfare effect of a marginal price change

on agent i (at initial wealth a_i) is

$$\frac{dU_i}{dr} = c_i^{-\sigma} \left[a_i + z_i \frac{dw}{dr} \right].$$

From the production side, $w = (1 - \alpha)ZK^\alpha$ and $(r + \delta) = \frac{\alpha Y}{\mu K}$, so that $dw/dr = -\alpha Y / (r + \delta) = -\mu K$. Using the household first-order condition $c_i^{-\sigma} = a_i^{-\Sigma}$:

$$\frac{dU_i}{dr} = a_i^{-\Sigma} (a_i - \mu K z_i).$$

Monotonicity of $\Phi(a) \equiv a^{-\Sigma}(a - \mu K z(a))$. Substituting $z(a) = (a^\phi - ar)/w$ from the budget constraint ($c = a^\phi$ and $c = ar + wz$, with $\phi \equiv \Sigma/\sigma$):

$$\Phi(a) = \left(1 + \frac{\mu Kr}{w} \right) a^{1-\Sigma} - \frac{\mu K}{w} a^{\phi-\Sigma}.$$

The exponents are $1 - \Sigma > 0$ (since $\Sigma < 1$) and $\phi - \Sigma = \Sigma(1 - \sigma)/\sigma < 0$ (since $\sigma > 1$). Differentiating:

$$\Phi'(a) = \underbrace{(1 - \Sigma) \left(1 + \frac{\mu Kr}{w} \right) a^{-\Sigma}}_{>0} + \underbrace{\frac{\mu K}{w} \cdot \frac{\Sigma(\sigma - 1)}{\sigma} \cdot a^{\phi-\Sigma-1}}_{>0} > 0,$$

since both terms are strictly positive. Therefore $a_1 > a_2$ implies $\Phi(a_1) > \Phi(a_2)$, i.e., $dU_1/dr > dU_2/dr$. Since $dr < 0$:

$$\Delta U_2 = \frac{dU_2}{dr} dr > \frac{dU_1}{dr} dr = \Delta U_1.$$

Full redistribution at $\mu = 1$. Under perfect competition, $V = 0$ so $A = K$. From step 3(i) of Proposition 1, $a_1/z_1 > K > a_2/z_2$, and therefore:

- $a_1 - Kz_1 > 0$, so $dU_1/dr > 0$: the rich loses welfare when r falls.
- $a_2 - Kz_2 < 0$, so $dU_2/dr < 0$: the poor gains welfare when r falls.

Hence the price effect is fully redistributive: $\Delta U_1 < 0 < \Delta U_2$.

A.5 Proof of Proposition 4, Items 1 and 3

To show that an increase in markups increases the equilibrium interest rate, we totally differentiate the equilibrium condition $A(r, \mu) = q(r, \mu)$ and obtain

$$dr^\mu = \frac{\varepsilon^{q,\mu} - \varepsilon^{A,\mu}}{\varepsilon^A + \varepsilon^q} d\mu > 0,$$

where $\varepsilon^{A,\mu} \equiv \partial \log A / \partial \mu < 0$ is the partial-equilibrium effect of markups on aggregate savings through the wage, and $\varepsilon^{q,\mu} \equiv \partial \log q / \partial \mu$ the partial-equilibrium effect of markups on the value of firms. This is positive as long as $\varepsilon^{q,\mu} > \varepsilon^{A,\mu}$, which we assume.

We maintain $\beta = 0$ throughout, and evaluate locally at $\mu = 1$.

Preliminaries: $dr/d\mu > 0$.

We first show that, in partial equilibrium (holding r constant), an increase in μ increases q . Write gross monopoly rents as $\Pi = \alpha \frac{\mu-1}{\mu} Y$. Then

$$\frac{\partial q}{\partial \mu} = \frac{\partial K}{\partial \mu} + \frac{1}{r} \frac{\partial \Pi}{\partial \mu}.$$

For the rent term, using the product rule:

$$\begin{aligned} \frac{\partial \Pi}{\partial \mu} &= \alpha Y \cdot \frac{1}{\mu^2} + \alpha \frac{\mu-1}{\mu} \cdot \frac{\partial Y}{\partial \mu} \\ &= \frac{\alpha Y}{\mu^2} - \alpha \frac{\mu-1}{\mu} \cdot \frac{\alpha Y}{(1-\alpha)\mu} \\ &= \frac{\alpha Y}{\mu^2} \left(1 - \frac{\alpha(\mu-1)}{1-\alpha} \right) = \frac{\alpha Y}{\mu^2} \cdot \frac{1-\alpha\mu}{1-\alpha}. \end{aligned}$$

Combining and substituting $\alpha Y = \mu(r + \delta)K$:

$$\frac{\partial q}{\partial \mu} = \frac{K}{(1-\alpha)\mu r} (\delta - \alpha\mu(r + \delta)) = \frac{K(r + \delta)}{(1-\alpha)\mu r} \left(\frac{\delta}{r + \delta} - \alpha\mu \right).$$

Hence $\partial q / \partial \mu > 0$ if and only if $\mu < \delta / [\alpha(r + \delta)]$, which is satisfied at $\mu = 1$ for standard calibrations. Evaluating at $\mu = 1$, where $V = 0$ so that $q = K$:

$$\varepsilon^{q,\mu} \Big|_{\mu=1} \equiv \frac{1}{q} \frac{\partial q}{\partial \mu} \Big|_{\mu=1} = \frac{\delta}{r} - \frac{\alpha}{1-\alpha}. \quad (10)$$

Effect of μ on savings supply. A change in μ also affects aggregate savings at fixed r through the wage. From the firm FOC, $\partial w / \partial \mu|_r = -\alpha w / [(1 - \alpha)\mu] < 0$. With $\beta = 0$, the steady-state Euler equation gives $c_i = a_i^\phi$ where $\phi \equiv \Sigma / \sigma \in (0, 1)$. Combined with the budget constraint, $a_i^\phi - a_i r = w z_i$. Applying the IFT:

$$\left. \frac{\partial a_i}{\partial \mu} \right|_r = \frac{z_i}{D_i^*} \cdot \left. \frac{\partial w}{\partial \mu} \right|_r = -\frac{\alpha w z_i}{(1 - \alpha)\mu D_i^*},$$

where $D_i^* \equiv \phi a_i^{\phi-1} - r > 0$. Therefore

$$\varepsilon^{A,\mu} \equiv \left. \frac{\partial \log A}{\partial \mu} \right|_r = -\frac{\alpha w}{(1 - \alpha)\mu A} \sum_i \frac{z_i}{D_i^*} < 0. \quad (11)$$

At $\mu = 1$, using $\alpha w / [(1 - \alpha)A] = \alpha Y / A = (r + \delta)$:

$$\left| \varepsilon^{A,\mu} \right|_{\mu=1} = (r + \delta) \sum_i \frac{z_i}{D_i^*}. \quad (12)$$

Totally differentiating $A(r, \mu) = q(r, \mu)$:

$$\frac{dr}{d\mu} = \frac{\varepsilon^{q,\mu} - \varepsilon^{A,\mu}}{\varepsilon^A + \varepsilon^q} > 0, \quad (13)$$

since both $\varepsilon^{q,\mu} > 0$ and $-\varepsilon^{A,\mu} > 0$.

1. ε^q increases with μ .

A change in μ affects ε^q both directly (through the firm valuation) and indirectly (through the equilibrium interest rate):

$$\frac{d\varepsilon^q}{d\mu} = \underbrace{\left. \frac{\partial \varepsilon^q}{\partial \mu} \right|_r}_{\text{direct}} + \underbrace{\frac{\partial \varepsilon^q}{\partial r} \cdot \frac{dr}{d\mu}}_{\text{indirect}}. \quad (14)$$

Recall that $\varepsilon^q = \varepsilon^K + \delta(\mu - 1) / (r\Delta)$ where $\Delta \equiv \mu r + (\mu - 1)\delta$. Differentiating and evaluating at $\mu = 1$:

$$\left. \frac{\partial \varepsilon^q}{\partial \mu} \right|_r = \frac{\delta}{r^2} > 0, \quad \frac{\partial \varepsilon^q}{\partial r} = -\frac{\varepsilon^K}{r + \delta} < 0.$$

The direct effect is positive: higher markups raise firm valuations, increasing the sensitivity of asset supply to the interest rate. The indirect effect is negative: higher markups raise

the equilibrium r , which reduces ε^q . Substituting the corrected $dr/d\mu$ from (13):

$$\frac{d\varepsilon^q}{d\mu} = \frac{\delta}{r^2} - \frac{\varepsilon^K}{r + \delta} \cdot \frac{\varepsilon^{q,\mu} - \varepsilon^{A,\mu}}{\varepsilon^K + \varepsilon^A}.$$

Since $\varepsilon^K + \varepsilon^A > \varepsilon^K$, the indirect term is bounded above (in absolute value) by its value at $\varepsilon^A = 0$:

$$\frac{d\varepsilon^q}{d\mu} > \frac{\delta}{r^2} - \frac{\varepsilon^{q,\mu} - \varepsilon^{A,\mu}}{r + \delta} = \frac{1}{r + \delta} \left(\frac{\delta^2}{r^2} + \frac{\alpha}{1 - \alpha} - |\varepsilon^{A,\mu}| \right). \quad (15)$$

This is positive provided

$$|\varepsilon^{A,\mu}| < \frac{\delta^2}{r^2} + \frac{\alpha}{1 - \alpha} \equiv \bar{\varepsilon}. \quad (16)$$

The threshold $\bar{\varepsilon}$ is of order $(\delta/r)^2$. For standard calibrations ($\delta = 0.1, r = 0.03, \alpha = 1/3$), $\bar{\varepsilon} \approx 11.6$, while $|\varepsilon^{A,\mu}| = (r + \delta) \sum_i z_i / D_i^*$ is of order $r + \delta \approx 0.13$. Condition (16) is therefore amply satisfied.¹

2. ε^A increases with μ .

The total change in ε^A along the equilibrium path decomposes as

$$\frac{d\varepsilon^A}{d\mu} = \underbrace{\frac{\partial \varepsilon^A}{\partial \mu} \Big|_r}_{\text{cross-partial}} + \underbrace{\frac{d^2 \log A}{dr^2} \cdot \frac{dr}{d\mu}}_{\text{convexity}}. \quad (17)$$

The convexity term is a product of two positive factors. We prove $d^2 \log A / dr^2 > 0$.

With $\beta = 0$, the budget equation is $a_i^\phi - a_i r = w z_i$. Define $D_i^* \equiv \phi a_i^{\phi-1} - r > 0$. The IFT gives $da_i/dr = \eta_i / D_i^*$, where $\eta_i \equiv a_i - K z_i$. Differentiating once more (using $dD_i^*/dr = \phi(\phi - 1)a_i^{\phi-2} a_i' - 1$ and $d\eta_i/dr = a_i' + K \varepsilon^K z_i$),

$$\frac{d^2 a_i}{dr^2} = \frac{2\eta_i}{(D_i^*)^2} + \frac{K \varepsilon^K z_i}{D_i^*} + \frac{\phi(1 - \phi) a_i^{\phi-2} \eta_i^2}{(D_i^*)^3}. \quad (18)$$

The last term is non-negative because $\phi(1 - \phi) > 0$: the concavity of the non-homothetic consumption function $c = a^\phi$ generates convexity in the savings response. At $\mu = 1, K = A$ so $\eta_1 + \eta_2 = 0$; write $\eta \equiv \eta_1 > 0$. Each summed component of $d^2 A / dr^2 = \sum_i d^2 a_i / dr^2$ is positive:

¹For (16) to fail would require $\min_i D_i^* < (r + \delta)r^2/\delta^2$, an extremely tight near-violation of the regularity condition $D_i^* > 0$.

- (i) $2\eta[(D_1^*)^{-2} - (D_2^*)^{-2}] > 0$, since $a_1 > a_2$ and $\phi - 1 < 0$ imply $D_1^* < D_2^*$.
- (ii) $K\varepsilon^K [z_1/D_1^* + z_2/D_2^*] > 0$.
- (iii) $\phi(1 - \phi)\eta^2 [a_1^{\phi-2}/(D_1^*)^3 + a_2^{\phi-2}/(D_2^*)^3] > 0$.

It remains to show the stronger result $d^2 \log A / dr^2 > 0$, i.e. $A \cdot d^2 A / dr^2 > (dA/dr)^2$. Since $(dA/dr)^2 = \eta^2 (D_2^* - D_1^*)^2 / [(D_1^*)^2 (D_2^*)^2]$, term (i) alone suffices:

$$A \cdot (i) - (dA/dr)^2 = \frac{\eta(D_2^* - D_1^*)}{(D_1^*)^2 (D_2^*)^2} [2A(D_1^* + D_2^*) - \eta(D_2^* - D_1^*)].$$

The bracket is positive: $2A > \eta$ (since $\eta = a_1 - Az_1 < Az_2 < A$) and $D_1^* + D_2^* > D_2^* - D_1^*$ (since $D_1^* > 0$). Terms (ii) and (iii) only add to the surplus. Therefore $d^2 \log A / dr^2 > 0$.

The cross-partial $\partial \varepsilon^A / \partial \mu|_r$ captures how the *slope* of the savings function shifts when markups change at fixed r . Its sign is ambiguous in general. However, $d\varepsilon^A / d\mu > 0$ whenever the convexity term dominates:

$$\left| \frac{\partial \varepsilon^A}{\partial \mu} \right|_r < \frac{d^2 \log A}{dr^2} \cdot \frac{dr}{d\mu}. \quad (19)$$

The cross-partial is of order $r + \delta$ (the same scale as $|\varepsilon^{A,\mu}|$), while the convexity term is of order $(\delta/r)^2 \cdot dr/d\mu$ (driven by the dominant term (i) in $d^2 A / dr^2$). Condition (19) is satisfied under the same parameter regime as (16).

3. $d \log(q/Y)$ increases with μ .

We have $d \log(q/Y) = \frac{N}{D} \partial \log A$ where $N \equiv \varepsilon^q - \varepsilon^Y$ and $D \equiv \varepsilon^A + \varepsilon^q$. From Steps 1–2, $dD/d\mu > 0$. For the numerator:

$$\frac{dN}{d\mu} = \frac{d\varepsilon^q}{d\mu} - \frac{d\varepsilon^Y}{d\mu}.$$

Since $\varepsilon^Y = \alpha \varepsilon^K = \alpha / [(1 - \alpha)(r + \delta)]$ and $dr/d\mu > 0$:

$$\frac{d\varepsilon^Y}{d\mu} = -\frac{\alpha}{(1 - \alpha)(r + \delta)^2} \cdot \frac{dr}{d\mu} < 0.$$

Hence $dN/d\mu > d\varepsilon^q/d\mu > 0$: the numerator grows even faster than ε^q alone. Since $dN/d\mu = d\varepsilon^q/d\mu + |d\varepsilon^Y/d\mu|$ and $dD/d\mu = d\varepsilon^q/d\mu + d\varepsilon^A/d\mu$, we have $dN/d\mu > dD/d\mu$

whenever

$$|d\varepsilon^Y/d\mu| > d\varepsilon^A/d\mu. \quad (20)$$

At $\mu = 1$, $N = (1 - \alpha)\varepsilon^K < \varepsilon^K + \varepsilon^A = D$. Combining $N < D$ with $dN/d\mu > dD/d\mu > 0$ yields $\frac{dN/d\mu}{N} > \frac{dN/d\mu}{D} > \frac{dD/d\mu}{D}$, so the ratio N/D increases.

Condition (20) requires that the decline in the output semi-elasticity ε^Y (a production-side object of order $\alpha\varepsilon^K \cdot dr/d\mu / (r + \delta)$) exceeds the increase in the savings semi-elasticity ε^A . Since $|d\varepsilon^Y/d\mu|$ scales with $\alpha(\varepsilon^K)^2 \cdot dr/d\mu \sim (\delta/r)^2$, while $d\varepsilon^A/d\mu$ is bounded by the convexity term (of the same order) plus a smaller cross-partial, this condition is satisfied under the same parameter regime as (16)–(19).

A.6 Proof of Proposition 4, Item 2

Part 1: $|(\varepsilon^{a_1} - \varepsilon^A)dr|$ decreases with μ (holding $\varepsilon^{a_1} - \varepsilon^A$ constant).

From the definition of the trickle-down effect,

$$|(\varepsilon^{a_1} - \varepsilon^A)dr| = \frac{(\varepsilon^{a_1} - \varepsilon^A) \partial \log A}{\varepsilon^A + \varepsilon^q}.$$

Since $\partial \log A$ is held fixed and $\varepsilon^A + \varepsilon^q$ increases with μ (Proposition 4), $|(\varepsilon^{a_1} - \varepsilon^A)dr|$ decreases with μ to first order (holding $\varepsilon^{a_1} - \varepsilon^A$ constant).

Part 2: Ω decreases with μ (under $\beta = 0$, at $\mu = 1$).

We maintain $\beta = 0$, $F = 0$ and evaluate locally at $\mu = 1$. Decompose the total derivative as

$$\frac{d\Omega}{d\mu} = \underbrace{\frac{\partial \Omega}{\partial \mu} \Big|_r}_{\text{direct}} + \underbrace{\frac{\partial \Omega}{\partial r} \Big|_\mu \cdot \frac{dr}{d\mu}}_{\text{indirect}}.$$

We show that the indirect effect is negative and dominates.

Step 1: Closed form. With $\beta = 0$, the Euler equation gives $c_i = a_i^\phi$ where $\phi \equiv \Sigma/\sigma \in (0, 1)$, and the budget constraint reads $a_i^\phi - a_i r = w z_i$. Define $D_i^* \equiv \phi a_i^{\phi-1} - r > 0$. As in Proposition 4, the total derivative of individual savings with respect to r is $da_i/dr = \eta_i/D_i^*$ where $\eta_i \equiv a_i - K z_i$. At $\mu = 1$, $K = A$ (since $V = 0$), so $\eta_1 + \eta_2 = 0$; write $\eta \equiv \eta_1 > 0$. The

log semi-elasticities are

$$\varepsilon^{a_1} = \frac{\eta}{a_1 D_1^*}, \quad \varepsilon^A = \frac{\eta(D_2^* - D_1^*)}{AD_1^* D_2^*},$$

where $D_2^* > D_1^*$ since $a_1 > a_2$ and $\phi < 1$ imply $a_1^{\phi-1} < a_2^{\phi-1}$. Taking the ratio:

$$\Omega = \frac{AD_2^*}{a_1(D_2^* - D_1^*)}. \quad (21)$$

Step 2: $d \log \Omega / dr < 0$ at fixed μ . Differentiating (21) logarithmically with respect to r :

$$\frac{d \log \Omega}{dr} = \underbrace{(\varepsilon^A - \varepsilon^{a_1})}_{<0} + \underbrace{\frac{D_1^{*'} D_2^* - D_2^{*'} D_1^*}{D_2^* (D_2^* - D_1^*)}}_{<0}, \quad (22)$$

where $D_i^{*'} \equiv dD_i^* / dr = \phi(\phi - 1)a_i^{\phi-2} \eta_i / D_i^* - 1$. The first term is negative because $\Omega > 1$ (Proposition 2). For the second, the cross-product evaluates to

$$D_1^{*'} D_2^* - D_2^{*'} D_1^* = \underbrace{\phi(\phi - 1)}_{<0} \eta \left[\frac{a_1^{\phi-2} D_2^*}{D_1^*} + \frac{a_2^{\phi-2} D_1^*}{D_2^*} \right] - \underbrace{(D_2^* - D_1^*)}_{>0}, \quad (23)$$

which is strictly negative since both terms have the same sign. Therefore $d \log \Omega / dr < 0$.

Step 3: Indirect effect dominates. Since $d \log \Omega / dr < 0$ and $dr / d\mu > 0$, the indirect effect reduces Ω . The direct effect $\partial \Omega / \partial \mu|_r$ captures the wage contraction at fixed r and is bounded by a term of order $r + \delta$, while the indirect effect is amplified by the equilibrium adjustment $dr / d\mu \propto \delta / r$. Under standard calibrations (and the same parameter regime as condition (16)), the indirect effect dominates, so that $d\Omega / d\mu < 0$.

Implication for the trickle-down effect. The trickle-down effect can be decomposed as

$$|(\varepsilon^{a_1} - \varepsilon^A) dr| = \underbrace{(\Omega - 1)}_{\text{excess sensitivity}} \cdot \underbrace{\frac{\varepsilon^A}{\varepsilon^A + \varepsilon^q}}_{\equiv s} \cdot \partial \log A,$$

where s denotes the demand share of asset-market clearing. As μ increases: (i) $\Omega - 1$ decreases (shown above); (ii) s decreases because the valuation channel makes ε^q grow proportionally faster than ε^A —at $\mu = 1$, the direct effect on ε^q is δ / r^2 (a large term driven by capitalized rents), while $d\varepsilon^A / d\mu$ is of order $r + \delta$ (the convexity term). Both factors

decline, so the overall trickle-down effect weakens.

B Additional Quantitative Results

Table B.1: Change in macro variables between initial and terminal steady states

Variable	$\mu = 1$	$\mu = 1.3$	$\mu = 1.45$
r	-1.1	-0.7	-0.6
w	6.3	4.1	3.7
η	17.3	21.4	21.4
Y	6.3	4.1	3.7
K	20.1	12.7	11.6
q	20.1	23.8	23.6
G	9.6	8.5	8.4

Note: This table shows the change in key macro variables between the initial (1970) and terminal steady states. The interest rate r is reported as a percentage point (p.p.) change. All other variables are reported as percentage (%) changes.

C Solution Method

Household problem. We solve the household’s dynamic programming problem using the endogenous grid method (EGM) of Carroll (2005). Because the wealth distribution exhibits a Pareto tail, we discretize the asset space on a non-linear grid of $n_A = 3,000$ points with curvature chosen to concentrate points in the empirically relevant range while extending the upper bound to $\bar{a} = 10^{12}$. We verify ex-post that the mass of the stationary distribution at the upper bound is negligible (below 10^{-10}), ensuring that the grid does not artificially truncate the tail.

Steady state. At the steady state, equity prices satisfy the no-arbitrage condition $q = (1 - \tau)\pi/r$, reducing the general equilibrium problem to a single unknown. To calibrate the initial steady-state, we solve for the discount factor β such that the asset market clears: $A = q + B$. For the terminal steady-state (with high-inequality), we find the level of

physical capital K that clears the asset market. The outer loop uses Brent’s method, while each evaluation solves the full household problem via EGM.

One-time transition. We compute perfect-foresight transitions following an unexpected, permanent MIT shock to the distribution of permanent labor income. Given initial and terminal steady states, we solve for the T -period path of the capital stock $\{K_t\}_{t=0}^{T-1}$ that clears the asset market at every date. We initialize the Jacobian of the asset market clearing condition with respect to K using the sequence-space Jacobian of Auclert et al. (2021), and then update it using Broyden’s method. At each iteration, given a candidate $\{K_t\}$, we compute the implied price sequences $\{r_t, w_t, q_t\}$ from the firm and arbitrage blocks, solve the household problem forward from the initial distribution, and evaluate the asset market residual. The Broyden update avoids recomputing the full $T \times T$ Jacobian at each step, making the algorithm tractable for long horizons ($T = 500$).

Myopic transition. The myopic transition models the gradual increase in permanent labor income inequality as a sequence of unanticipated MIT shocks. At each date $t \in \{1, \dots, T_{\text{trans}}\}$, the economy is hit by a new shock to the permanent income distribution $\{z_i\}_t$, and households treat it as one-time and permanent. Conditional on the shock, they have perfect foresight over future prices. Formally, at date t we solve a full perfect-foresight transition from the current state of the economy to the new terminal steady state implied by $\{z_i\}_t$. We then extract the period- t outcomes—prices, distributions, and aggregates—and use the resulting distribution D_{t+1} as the initial condition for the next shock at $t + 1$. This nests a sequence of MIT-shock transitions, each solved using the Broyden algorithm described above.

References

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