Labor Income Inequality and the Transmission of Monetary Policy

Eustache Elina[*] Raphaël Huleux[†]

September 2024

Abstract

We study the impact of an increase in permanent labor income inequality on the transmission of monetary shocks on the real economy. In a Heterogeneous-Agent New-Keynesian model with standard preferences, we show that the distribution of permanent labor income is neutral with respect to monetary policy shocks. However, this model cannot account for the observed relationship between permanent income and consumption-saving behavior. Including a non-homothetic taste for wealth allows us to match this relationship, and breaks the neutrality result. The direct substitution effect from a monetary policy shock is weakened while indirect effects are stronger. The rise in permanent labor income inequality makes households hold wealth more for a present motive rather than for an intertemporal-substitution motive. As a result, the aggregate elasticity of intertemporal substitution is weakened while the aggregate static MPC is strengthened. In a realistic two-asset HANK model, we quantify the change in the composition of a monetary shock. We observe a rise in the magnitude of a monetary policy shocks as the increase in indirect effects more than outweighs the fall in the direct effect.

Keywords: Inequality, permanent income, monetary policy, interest rates, heterogenous agent model.

JEL codes: D63, E24, E52, E4

[*]Paris School of Economics, eustache.elina@psemail.eu

[†]Copenhagen Business School, rah.eco@cbs.dk

This work would not have been possible without the fruitful discussions with Leonard Bocquet who shares the starting idea of the paper. Our work also largely benefited from the helpful supervision and guidance of Axelle Ferrière, Tobias Broer, David Jinkins, Birthe Larsen, and Karl Harmenberg. We also thank Edouard Challe, Xavier Ragot, Riccardo Cioffi, Julien Matheron, Alex Clymo, Charles Labrousse, and Yann Perdereau for their helpful comments. All remaining errors are our own.

Introduction

In the US, labor income inequality has increased steadily since the 1980s, with the share of pre-tax total labor income earned by the top 1% doubling from 6% in 1980 to 12% in 2020 (Saez & Zucman 2020). Existing empirical literature suggests that the surge in labor income inequality is predominantly attributable to an increase in the *permanent* component of labor income inequality (DeBacker et al. 2011, Bloom et al. 2017, Braxton et al. 2021, Guvenen et al. 2022)) —a term we use to denote the variance in initial outcomes for new cohorts entering the labor market (Straub 2019). In this paper, we explore the implications of rising permanent labor income inequality for monetary policy. We ask two questions. What is its effect on the strength of a monetary policy shock? How does it change the transmission channels of monetary policy?

In a simple textbook IS-LM model, an increase in permanent labor income inequality decreases the output response to monetary policy, if rich households have a lower marginal propensity to consume than poorer ones. Indeed, assume two households – a low income and a high income – with population shares ω and $(1-\omega)$. The low type receives a share $1-z$ of total income and has a high marginal propensity to consume m_l while the high type receives a share z and has a low marginal propensity to consume $m_h^y < m_l^y$. The static Keynesian cross writes[1]

$$dY = \frac{1}{1-\omega(1-z)m_l^y - (1-\omega)zm_h^y} b_1 dr.$$

An increase in permanent labor income inequality z will thus redistribute income towards low MPCs households, decreasing the multiplier, and hence the output response to a monetary policy shock. This simple model is useful to motivate why the distribution of permanent income might matter for monetary policy, but it misses a few core features of modern macroeconomic models: it lacks dynamics and ignores the endogeneity of the consumption-savings decision to permanent income.

In this paper, we explore this question using a HANK model with rich household-level heterogeneity and featuring an intertemporal Keynesian cross à la Auclert et al. (2024). We introduce a decomposition of the impact of rising permanent labor income inequality on monetary policy between three channels. First, an increase in permanent income inequality

[1] b_1 is the sensitivity of aggregate consumption to the interest rate.

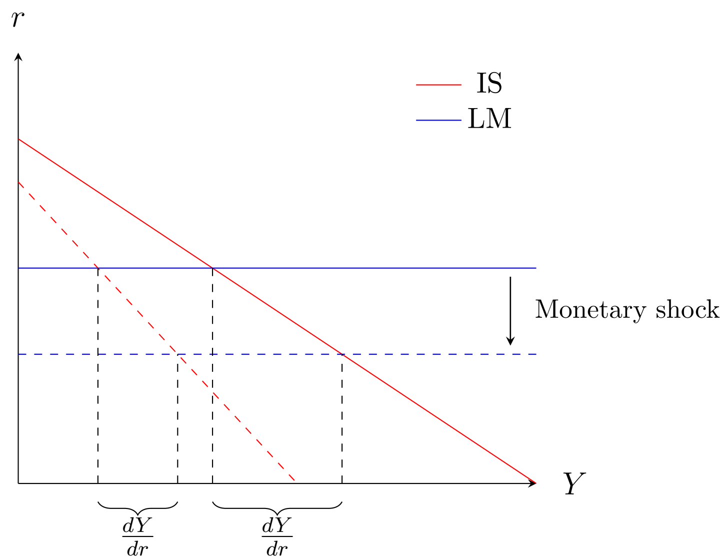

Figure 1: Change in equilibrium output in an IS-LM model after a monetary policy shock

Note: The red full line shows the IS curve with low inequality, while the red dashed curve shows the IS curve with high inequality.

redistributes income between households with potentially different responses to income and interest rate shocks. We call this channel the composition effect, which is the only channel present in the simple IS-LM model we presented. Secondly, households might further change their response to shocks as they observe an increase or a decrease in their permanent income. We call this the policy function effect. Finally, as the behavior of households changes, their position in the distribution of wealth might change as well, changing the aggregate reaction to interest and income shocks. We call this channel the wealth distribution effect.

Our paper starts with a neutrality result: in a model with preferences only on consumption, those three channels are exactly equal to zero. In this setting, consumption is a linear function of permanent income. As a result, whatever the distribution of permanent income, the aggregate response of consumption following a monetary shock will remain the same, consistently with Straub (2019). This neutrality result is useful as a benchmark, but largely unrealistic. Indeed, a large empirical literature has shown that households with higher levels of permanent income have a higher marginal propensity to save out of permanent income than the rest of the distribution (Carroll 1998, Dynan et al. 2004, Kumhof et al. 2015, Straub 2019, Mian et al. 2020).

A growing literature has been solving that issue by adding wealth to the utility function as a luxury good (Kumhof et al. 2015, Straub 2019, Mian et al. 2020). By doing so, they are able to match the relationship between consumption-saving decisions and the level of permanent income. We show that this type of preference breaks down our neutrality result and ex-ante heterogeneity in permanent income matters for the output response to a monetary shock. The rise in permanent labor income inequality observed in the US from 1989 to 2019 changes the transmission channels of monetary policy and raises the output elasticity to an interest shock by 12.5%.

We first study the effect of a rise in permanent labor income inequality in a zero-liquidity HANK model. By doing so, we are able to analytically characterize the contribution of the composition effect and the policy function effect on the transmission channels of monetary policy and its aggregate effect. We show that, at the household level, a rise in permanent labor income raises the sensitivity to an income shock – the marginal propensity to consume – while it dampens the sensitivity to an interest rate shock – the elasticity of intertemporal substitution.

In order to capture the wealth distribution effect, we then relax the zero-liquidity assumption. We show that the rise in permanent labor income inequality decreases the relative share of the direct effect – the effect of a change in the interest rate on consumption, keeping household disposable labor income constant – while raising the share of indirect effects – the effect of the change in household disposable income keeping the path of the interest rate constant. Indeed, the rise in permanent labor income inequality pushes the equilibrium real interest down which increases the income share going to hand-to-mouth households. The increase in the general equilibrium multiplier raises the aggregate effect of a monetary policy shock.

The paper concludes that a simple application of the IS-LM model is misguided: an increase in permanent labor income inequality increases the output response to a monetary policy shock through a rise in the general equilibrium multiplier.

Literature. Our paper belongs to the old and large literature that investigates the transmission channels of monetary policy on aggregate variables. Over the last two decades, this literature has gradually moved away from a representative-agent framework to explore the interactions between distributions of income and wealth, and monetary policy. The wealth distribution matters for monetary policy as it determines the share of direct and indirect effects in the transmission of shocks. Micro survey data on household portfolio reveals that a large portion of households do not hold liquid wealth (Kaplan et al. 2014). Those households do not react to interest rate changes but are very sensitive to changes in their disposable income. When including that heterogeneity, Kaplan et al. (2018) shows that most of the transmission of monetary policy goes through indirect general equilibrium effects (mainly through the change in labor demand). The composition of a monetary shock matters for policymakers as Holm et al. (2021) shows that indirect effects take time to materialize. As a result, if most of the effect of monetary policy goes through indirect effects, monetary policy is not capable of raising aggregate output in a short time frame.

Monetary policy has also a feedback effect on the wealth and the income distributions (Coibion et al. 2017, Mumtaz & Theophilopoulou 2017). Those papers show that contractionary monetary policy is usually procyclical while expansionary monetary policy tends to dampen income inequality. This feedback effect from the monetary policy on the income distribution not only matters from an equity standpoint but also since it is one of the transmission channels of monetary policy on aggregate variables. Indeed, by redistributing income from low-MPC households to high-MPC households, expansionary monetary policy is amplified compared to the representative-agent benchmark (Auclert 2019).

While the interactions between wealth distribution and monetary policy have been extensively studied, the way the distribution of labor income shapes monetary policy has remained partially under the radar. The main reason is that the drivers behind the increase in labor income inequality were not well understood until recently. The macro implication of a rise in the variance of shocks over the lifetime is vastly different from the rise in the variance of initial outcomes. Both Bloom et al. (2017) and Braxton et al. (2021) show that temporary earning risks have declined. DeBacker et al. (2011) and Guvenen et al. (2022) show that most of the increase in labor income inequality has come from an increase in the variance of initial outcomes. Our contribution is to study the implications of that increase on the way monetary policy shocks are transmitted.

Our paper uses recent theoretical and methodological advances made by Auclert et al. (2024). They show that modern micro-founded heterogenous-agent New-Keynesian models feature an ”intertemporal Keynesian cross”, characterized by the sequence-space Jacobian of the consumption function. Our paper extends this methodology to monetary policy, and we recover a similar intertemporal Keynesian cross for monetary policy shocks.

Our paper also belongs to the growing literature that uses a non-homothetic taste for wealth to solve various puzzles. Carroll (1998) proposes wealth as a source of social status to explain why rich households have such high saving rates compared to the rest of the population. Straub (2019) shows that non-homothetic preferences capture the saving behavior along the distribution of permanent income and explain how variations in permanent labor income inequality increase wealth inequality and decrease the equilibrium interest rate. Kumhof et al. (2015) also uses a non-homothetic taste for wealth to match the saving behavior of richer households, in a model that captures the accumulation of debt in the U.S. in the years preceding the Great Recession. In their literature review, De Nardi et al. (2016) also mention non-homothetic bequest motive to explain why a large number of households die with significant amounts of wealth. Within the New-Keynesian literature, Michau (2018) and Mian et al. (2021) also include bonds in the utility to study the over-accumulation of savings in modern economies. Lastly, in Gaillard & Wangner (2021), the non-homothecity in the taste for wealth allows to match the thicker tail of the income distribution compared to the consumption distribution. This finding is consistent with the concavity in the consumption function with respect to permanent income generated by this non-homothecity.

Our emphasis on the interactions between non-homothetic preference in the saving behavior and monetary policy is shared by Michaillat & Saez (2021) and Melcangi & Sterk (2020). Consistent with Michaillat & Saez (2021), the presence of bonds-in-the-utility solves the forward guidance puzzle, even in the absence of idiosyncratic shocks. Melcangi & Sterk (2020) is also concerned by the change in the policy channels and in the aggregate effect of monetary policy. The paper also finds out that, in the US since 1980, the power of monetary policy has strengthened following the rise in the stock market participation channel.

In section 1, we study the impact of an increase in permanent labor income inequality in a one-asset HANK model. We focus on the limit case of zero-liquidity. By doing so, we are able to analytically characterize the effect of the permanent income distribution on the transmission of monetary policy. We then relax that assumption and show that the permanent labor income distribution also changes the transmission of monetary policy through a wealth distribution effect. In section 2, we calibrate a two-asset HANK model and measure quantitatively the effect of the change in the labor income distribution on the transmission of monetary policy and on its aggregate effect.

1 Permanent income inequality and monetary policy in HANK

This section studies the impact of a change in the distribution of permanent income in a heterogenous-agent New-Keynesian model, with and without non-homothetic preferences for wealth. We first focus on the zero-liquidity case to gain tractability. The stylized nature of the model allows us to characterize clearly when the distribution of permanent income is non-neutral with respect to the transmission of monetary policy. In the last section, we then relax that assumption by having positive liquidity.

1.1 Setup

Households. The economy is composed of a continuum of high-productivity households of mass $1-\omega$ denoted by h, and a continuum of low-productivity households of mass ω, denoted l. Households face idiosyncratic productivity shocks. They have access to financial markets and can smooth their consumption over time by holding government bonds. A household of permanent type $i \in \{l, h\}$ solves the following maximization problem:

$$\max_{\{c_{i,t}\}_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\frac{{c_{i,t}}^{1-\sigma} - 1}{1-\sigma} + \gamma \frac{(a_{i,t+1} + \zeta)^{1-\Sigma}}{1-\Sigma} \right),$$

subject to

$$c_{i,t} + a_{i,t+1} = (1-\tau_t) z_i e_{i,t} N_t + (1+r_t) a_{i,t} \text{ and } a_{i,t+1} \geq 0.$$

$a_{i,t}$ denotes the wealth position of the household at the beginning of period t, $c_{i,t}$ is the consumption choice, N_t is the labor supply, $e_{i,t}$ is the idiosyncratic productivity type, z_i is the level of permanent productivity, and τ_t is the linear tax rate. The labor supply of households is infinitely elastic. In this framework, an increase in labor income inequality means an increase in z_h. We normalize total productivity to 1 which implies that $\omega z_l + (1-\omega) z_h = 1$. Whenever the level of inequality increases, z_l also decreases to keep total productivity constant. The idiosyncratic productivity shock $e_{i,t}$ follows an AR-(1) process that we discretize using the Tauchen method.

We define the measure $\mu_{i,t}(a, e)$ as the mass of household of permanent type i holding wealth a with idiosyncratic type e at time t and $c_{i,t}(a, e)$ as the policy function for consumption for an agent holding wealth a with idiosyncratic type e at time t. Aggregate consumption is given by:

$$C_t = \omega \int c_{l,t}(a,e) d\mu_{l,t}(a,e) + (1-\omega) \int c_{h,t}(a,e) d\mu_{h,t}(a,e).$$

Firms. A representative firm uses labor to produce a final good according to the following production function $Y_t = (z_l\omega + z_h(1-\omega))N_t = N_t$. Firms are in perfect competition and subject to complete rigidity on the real wage (normalized to one). The representative firm maximizes its profits $Y_t - N_t$ subject to the technology and the demand constraint $Y_t = C_t$. Due to the wage rigidity, the firm is demand-constrained and any variation in effective demand C_t generates a one-to-one relationship with the labor demand N_t. This wage rigidity thus translates into a complete price rigidity.

Government. The government funds the service of public debt B with a linear tax rate τ_t on labor income so as to keep its budget balanced:

$$\tau_t N_t = r_t B.$$

Central bank. The central bank fixes an exogenous path for the nominal interest rate. As prices are rigid, any variations in the nominal interest rate translate into variations in the real interest rate r_t.

Definition 1. *A competitive equilibrium is defined as policy functions for consumption* $\{c_{i,t}(a,e)\}_{i,t}$ *and savings* $\{a_{i,t+1}(a,e)\}_{i,t}$*; sequences for labor demand* $\{N_t\}$*, the real interest rate* $\{r_t\}$*, the linear tax rate* $\{\tau_t\}$*, and output* $\{Y_t\}$*; a measure* $\{\mu_{i,t}(a,e)\}_{i,t}$ *such that :*

1. *Households solve their problem given prices and labor demand;*
2. *The representative firm maximizes profits;*
3. *The government maintains a balanced budget;*
4. *Markets clear:*
   (a) *The asset market clears:*
   $\omega \int a_{l,t+1}(a,e) d\mu_{l,t}(a,e) + (1-\omega) \int a_{h,t+1}(a,e) d\mu_{h,t}(a,e) = B, \quad \forall t$
   (b) *The goods market clears:*
   $Y_t = \omega \int c_{l,t}(a,e) d\mu_{l,t}(a,e) + (1-\omega) \int c_{h,t}(a,e), \quad \forall t$

1.2 The intertemporal Keynesian cross and permanent labor income inequality

The output response of this model following a monetary policy shock can be described by an intertemporal Keynesian cross as in Auclert & Rognlie (2020). Indeed, note that aggregate consumption at time $t = 0$ can be written as

$$C_0 \equiv \mathcal{C}_0(\{r_t, \tilde{N}_t\}_{t\geq 0})$$

where $\tilde{N}_t = (1-\tau_t)N_t$ is the net labor income and $\mathcal{C}_0(\{r_t, \tilde{N}_t\}_{t\geq 0})$ is the aggregate consumption policy function that depends on all the future paths of real interest rate and net labor income. We assume that the function $\mathcal{C}_t : \ell^\infty \times \ell^\infty \to \ell^\infty$ is Fréchet-differentiable. Totally differentiating the aggregate consumption function, we obtain

$$\begin{aligned} d\mathcal{C}_0(\{r_t, \tilde{N}_t\}_{t\geq 0}) = & \left[\frac{\partial C_0}{\partial r_0} dr_0 + \frac{\partial C_0}{\partial r_1} dr_1 + ...\right] \\ & + \left[\frac{\partial C_0}{\partial \tilde{N}_0} d\tilde{N}_0 + \frac{\partial C_0}{\partial \tilde{N}_1} d\tilde{N}_1 + ...\right]. \end{aligned}$$

We can rewrite this expression more succinctly by writing it in vector forms

$$d\mathbf{C}(\{r_t, \tilde{N}_t\}_{t\geq 0}) = \mathbf{M}^r d\mathbf{r} + \mathbf{M}^{\tilde{n}} d\tilde{\mathbf{N}},$$

where $\mathbf{M}^r$ and $\mathbf{M}^{\tilde{n}}$ are aggregate sequence-space Jacobians (infinite-dimensional matrices) which characterize the reaction of aggregate consumption facing shocks on the real interest rate and on net labor demand at different periods, as in Auclert et al. (2024, 2021):

$$\mathbf{M}^r \equiv \begin{pmatrix} \frac{\partial \mathcal{C}_0}{\partial r_0} & \frac{\partial \mathcal{C}_0}{\partial r_1} & \cdots \\ \frac{\partial \mathcal{C}_1}{\partial r_0} & \frac{\partial \mathcal{C}_1}{\partial r_1} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \quad \text{and } \mathbf{M}^{\tilde{n}} \equiv \begin{pmatrix} \frac{\partial \mathcal{C}_0}{\partial \tilde{N}_0} & \frac{\partial \mathcal{C}_0}{\partial \tilde{N}_1} & \cdots \\ \frac{\partial \mathcal{C}_1}{\partial \tilde{N}_0} & \frac{\partial \mathcal{C}_1}{\partial \tilde{N}_1} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}.$$

Note that we can also define the aggregate consumption function as the sum of the consumption functions for each permanent type *by unit of permanent income*:

$$\mathcal{C}_0 \equiv \omega z_l \mathcal{C}_{l,0} + (1-\omega) z_h \mathcal{C}_{h,0}.$$

The change in aggregate consumption is thus given by

$$\begin{aligned} d\mathcal{C} &= (1-\omega) z_h d\mathcal{C}_h + \omega z_l d\mathcal{C}_l \\ &= (1-\omega) z_h (\mathbf{M}_h^r d\mathbf{r} + \mathbf{M}_h^n d\tilde{\mathbf{N}}) + \omega z_l (\mathbf{M}_l^r d\mathbf{r} + \mathbf{M}_l^n d\tilde{\mathbf{N}}) \\ &= \underbrace{((1-\omega) z_h \mathbf{M}_h^r + \omega z_l \mathbf{M}_l^r) d\mathbf{r}}_{\text{direct effect}} + \underbrace{((1-\omega) z_h \mathbf{M}_h^n + \omega z_l \mathbf{M}_l^n) d\tilde{\mathbf{N}}}_{\text{indirect effect from net labor demand}}. \end{aligned}$$

With $\mathbf{M}_i^x$ the sequence-space jacobian of permanent type *i per unit of permanent income.* The aggregate-level sequence-space Jacobian can thus be written as the following weighted average:

$$\forall x \in \{r, \tilde{n}\}, \quad \mathbf{M}^x = \omega z_l \mathbf{M}_l^x + (1-\omega) z_h \mathbf{M}_h^x.$$

Imposing that, in general equilibrium, $d\mathbf{Y} = d\mathbf{N} = d\mathbf{C}$, $d\boldsymbol{\tau} = Bd\mathbf{r} - rBd\mathbf{N}$, and $d\tilde{\mathbf{N}} = (1-\tau)d\mathbf{N} - d\boldsymbol{\tau}$, we can compute the aggregate effect of a monetary shock $d\mathbf{r}$ which is given by Proposition 1.

Proposition 1. *The aggregate effect of a monetary shock* $d\mathbf{r}$ *is the product of the general equilibrium multiplier and of the direct effect:*

$$d\mathbf{Y} = \mathcal{M}(\mathbf{M}^r - B\mathbf{M}^{\tilde{n}})d\mathbf{r}, \tag{1}$$

with $\mathcal{M} = [\mathbf{K}(\mathbf{I} - \mathbf{M}^{\tilde{n})}]^{-1}\mathbf{K}$, $\mathbf{K} \equiv -\sum_{t=1}^{\infty}(1+r)^{-t}\mathbf{F}^t$, *and* $\mathbf{F}$ *the forward matrix.*

Proof. Appendix A.1 □

Proposition 1 shows the output response in our model as a function of the sequence space Jacobians. We can recover from this expression the traditional channels of monetary policy. $\mathbf{M}^r$ captures the direct effect of monetary policy, the change in consumption plans induced by a change in the real interest rate, keeping $\{\tilde{N}_t\}$ constant. A change in the path of the real interest rate has two direct effects: an intertemporal substitution effect and an income effect. By changing relative prices of consuming at different time periods, the new path of the real interest rate induces a change in consumption today. The change in the path of the real interest rate also changes households' financial income, implying a further change in consumption.

A change in the path of $\mathbf{r}$ also affects the budget constraint of the government, which will imply a direct change in the path of taxes τ, further changing the consumption path. This second effect is captured by the $-B\mathbf{M}^{\tilde{n}}$ in the IKC. The total change in the consumption path of households changes aggregate demand, which changes the firm's labor demand and hence the labor income of all households. This second change in consumption, in turn, has a feedback effect on aggregate demand, which is the traditional Keynesian multiplier effect. This general equilibrium amplification is the indirect effect from labor demand, and is captured in the IKC by the matrix $\mathcal{M}$, which is itself a function of the intertemporal marginal propensity to consume matrix $\mathbf{M}^{\tilde{n}}$.

Using the IKC as an ordering device, we ask: what happens when permanent labor income inequality goes up? We compare the output response following a monetary shock of two economies, one characterized by a low level of permanent labor income inequality ($\mathbf{z} = (z_l, z_h)$ while the other one is characterized by a higher level of permanent labor income inequality ($\mathbf{z}' = (z'_l, z'_h)$). A bold z indexes economies with different levels of permanent labor income inequality. Proposition 2 shows that we can decompose this change in the output response to a monetary shock between three effects: a composition effect, a Jacobian effect and an amplification effect.

Proposition 2. *The difference in the output response following a monetary policy shock in two economies with different levels of permanent income inequality can be decomposed between three terms: (1.) a direct and an indirect composition effect, (2.) a direct and an indirect Jacobian effect, and (3.) an amplification effect:*

$$
\begin{aligned}
d\boldsymbol{Y}(\mathbf{z}') - d\boldsymbol{Y}(\mathbf{z}) = {} & \underbrace{\overbrace{\left[(1-\omega)\Delta z'_h \boldsymbol{M}^r_h(\mathbf{z}') + \omega \Delta z'_l \boldsymbol{M}^r_l(\mathbf{z}')\right] d\boldsymbol{r}}^{\textit{direct}} + \overbrace{\left[(1-\omega)\Delta z'_h \boldsymbol{M}^{\tilde{n}}_h(\mathbf{z}') + \omega \Delta z'_l \boldsymbol{M}^{\tilde{n}}_l(\mathbf{z}')\right] d\tilde{\mathbf{N}}(\mathbf{z}')}^{\textit{indirect}}}_{\textit{composition effect}} \\
& + \underbrace{\overbrace{\left[(1-\omega) z_h \Delta \boldsymbol{M}^r_h(\mathbf{z}') + \omega z_l \Delta \boldsymbol{M}^r_l(\mathbf{z}')\right] d\boldsymbol{r}}^{\textit{direct}} + \overbrace{\left[(1-\omega) z_h \Delta \boldsymbol{M}^{\tilde{n}}_h(\mathbf{z}') + \omega z_l \Delta \boldsymbol{M}^{\tilde{n}}_l(\mathbf{z}')\right] d\tilde{\mathbf{N}}(\mathbf{z}')}^{\textit{indirect}}}_{\textit{Jacobian effect}} \\
& + \underbrace{\boldsymbol{M}^{\tilde{n}}(\mathbf{z})\left(d\tilde{\mathbf{N}}(\mathbf{z}') - d\tilde{\mathbf{N}}(\mathbf{z})\right)}_{\textit{amplification effect}}.
\end{aligned}
$$

With $\Delta \boldsymbol{M}^x(\mathbf{z}') = \boldsymbol{M}^x(\mathbf{z}') - \boldsymbol{M}^x(\mathbf{z})$ *and* $\Delta z'_i = z'_i - z_i$.

Noting that $d\boldsymbol{Y}(\boldsymbol{z}') - d\boldsymbol{Y}(z) = d\boldsymbol{N}(\boldsymbol{z}') - d\boldsymbol{N}(\mathbf{z})$, *we can write*

$$
d\mathbf{Y}(\mathbf{z}') - d\mathbf{Y}(\mathbf{z}) = \mathcal{M}(\textit{composition effect} + \textit{Jacobian effect}).
$$

Proof. Appendix A.2 □

The composition effect captures the fact that even if the response of consumption *per unit of permanent income* for each type does not change, the increase in inequality will put more weight on the response of the high type compared to the low type.

The Jacobian effect captures the change in the coefficients of our sequences-space Jacobians keeping the weights on each matrices constant. The change in coefficients comes from three factors: (1) both types change their consumption-saving decisions as their income increases, (2) in general equilibrium prices adjust, and this change in prices also affects the consumption-saving decisions of households, and (3) the distribution of households also changes across the two economies.

More formally, the change in the type-i Jacobian's now depends on the change in the policy function *and* in the distribution of wealth $\mu_{i,t}$. Indeed, the definition of $M^x_{i,t,j}$, the element in line t, column j of the sequence-space Jacobian M^x_i, can be written as[2]

$$M^x_{i,t,j}(\mathbf{z}) = \int \frac{\partial c_{i,t}(a,e;\mathbf{z})}{\partial x_j} d\mu_{i,t}(a,e;\mathbf{z}).$$

Using this definition of a sequence-space Jacobian we can compute ΔM^x_i, the change in the Jacobian of an agent of type i in period t following a shock on the aggregate variable x in period j as

$$\Delta M^x_{i,t,j}(\mathbf{z}') = \underbrace{\int \left(\frac{\partial c_{i,t}(a,e;\mathbf{z}')}{\partial x_j} - \frac{\partial c_{i,t}(a,e;\mathbf{z})}{\partial x_j}\right) d\mu_{i,t}(a,e;\mathbf{z})}_{\text{policy-function effect (1)+(2)}} + \underbrace{\int \frac{\partial c_{i,t}(a,e;\mathbf{z})}{\partial x_j}\Big(d\mu_{i,t}(a,e;\mathbf{z}') - d\mu_{i,t}(a,e;\mathbf{z})\Big)}_{\text{wealth-distribution effect (3)}}$$

Finally, in general equilibrium, the amplification effect magnifies the first two effects. It acts a multiplier of the sum of the composition and the Jacobian effect. Thus, even small Jacobian and composition effect might end up having a large impact on the output response, since they will be amplified by the general equilibrium matrix $\mathcal{M}$. Both the composition and the Jacobian effects can be decomposed into a direct and an indirect effect.

1.3 HANK with Zero-Liquidity

In this section, we focus on the particular case of zero-liquidity $B = 0$ which allows us to compute analytically our sequence-space Jacobians. Using those analytical forms, we

[2]Note that the path of distribution $d\mu_t(a,e;s)$ itself depends on the derivative of the policy functions.

compute the aggregate effect of a monetary policy shock. We also perform a decomposition between the direct effect of the real interest rate shock and the indirect effect coming from the change in labor demand. In this framework, we can analytically characterize the effect of the distribution of permanent labor income as well as the effect of the taste for wealth on the transmission of a monetary policy shock.

1.3.1 Analytically solving the Sequence-Space Jacobians

We first derive the analytical expressions for the sequence space Jacobians that compose Equation A.1 under the zero-liquidity assumption. Note that those expressions do not depend on the particular preferences we assume in the household problem, and thus apply to both the case with homothetic preferences and non-homothetic preferences. We will distinguish the impact of those different preferences in the Sub-sections 1.3.3 and 1.3.4.

In the presence of zero liquidity, all households with an idiosyncratic type lower than the maximum idiosyncratic type $\bar{e}$ will be against the borrowing constraint. Those households behave as hand-to-mouth households and will not react to variations in the real interest rate, while their consumption will react one-to-one to variations in labor income. Defining the saving policy function of permanent type i, $a_i(a, e)$, we have that, at the steady state, $\frac{\partial a_i}{\partial a}(0, e) = 0, \ \forall e < \bar{e}$[3].

At the steady state, households with the highest idiosyncratic type $\bar{e}$ are on their Euler equation, and hence, are indifferent between savings and dissavings. The slope of the steady state saving function of a permanent type i is given by $\lambda_i \equiv \frac{\partial a_i}{\partial a}(0, \bar{e})/\Pi_{\bar{e}\bar{e}}$, with $\Pi_{\bar{e}\bar{e}}$ the probability of staying a high-idiosyncratic type tomorrow conditional on being a high-idiosyncratic type today[4]. For clarity, we define $1 - \mu \equiv \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}$ as the effective share of unconstrained households within permanent type i.

Proposition 3. *a) The sequence-space Jacobian of the consumption response following a*

[3]See appendix B.2 for formal derivation.

[4]See appendix B.3 for formal derivation.

labor demand shock of permanent income type i is

$$\mathbf{M}_i^{\tilde{n}} = \mu\mathbf{I} + (1-\mu)\begin{pmatrix} 1-\frac{\lambda_i}{1+r} & \frac{\lambda_i}{1+r}(1-\beta\lambda_i) & \cdots \\ \lambda_i\left(1-\frac{\lambda_i}{1+r}\right) & \left(1-\frac{\lambda_i}{1+r}\right)\Big(1-\lambda_i(1-\beta\lambda_i)\Big) & \cdots \\ \lambda_i^2\left(1-\frac{\lambda_i}{1+r}\right) & \lambda_i\left(1-\frac{\lambda_i}{1+r}\right)\Big(1-\lambda_i(1-\beta\lambda_i)\Big) & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

where $\boldsymbol{I}$ *is the matrix of intertemporal marginal propensities to consume (iMPCs) of constrained households and the second matrix is the iMPCs of unconstrained households of permanent type i.*

b) The sequence-space Jacobian of the consumption response following an interest rate shock of permanent income type i is

$$\boldsymbol{M}_i^r = (1-\mu)\beta\bar{e}\rho(\bar{e})\frac{1}{\sigma}\frac{\lambda}{\Pi_{\bar{e}\bar{e}}}\begin{pmatrix} 0 & -\frac{1}{1+r} & -\frac{\beta\lambda_i}{1+r} & \cdots \\ 0 & 1-\frac{\lambda_i}{1+r} & \beta\lambda_i(1-\frac{\lambda_i}{1+r})-\frac{1}{1+r} & \cdots \\ 0 & \lambda_i(1-\frac{\lambda_i}{1+r}) & (1+\beta\lambda_i^2)(1-\frac{\lambda_i}{1+r}) & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

where $\rho(\bar{e}) = \sum_{e'}\Pi_{\bar{e}e'}\left(\frac{e'}{\bar{e}}\right)^{-\sigma}$

Proof. For $\mathbf{M}_i^r$, see Appendix B.5 and for $\mathbf{M}_i^n$, see Appendix B.8 □

The interpretation of those two sequence-space Jacobian is intuitive.

Jacobian with respect to the labor demand. Starting with the sequence jacobian of the consumption response following a labor demand shock, the first column of $\mathbf{M}_i^{\tilde{n}}$ gives us the intertemporal marginal propensity to consume (iMPC) out of a one-time income shock of households of permanent type i. It is a weighted average of the iMPCs of the constrained and unconstrained households *among* a permanent type i. Note that the MPC of constrained households is 1, and that the MPC of unconstrained households is $1-\lambda_i/(1+r)$. In the absence of a taste for wealth and idiosyncratic shocks, $\lambda_i = 1$ and we recover the standard MPC in a representative agent model, $r/(1+r)$.

Aggregating over both permanent types, we obtain the aggregate sequence-space Jacobian $\mathbf{M}^{\tilde{n}} = \omega_l z_l \mathbf{M}_l^{\tilde{n}} + \omega_h z_h \mathbf{M}_h^{\tilde{n}}$, a weighted sum of the Jacobian per permanent-type. The contemporaneous aggregate iMPC[5] is given by the first element of the first column of this infinite matrix:

$$
\begin{aligned}
iMPC_0 &= 1-(1-\mu)\frac{1}{1+r}\sum_i \omega_i z_i \lambda_i \\
&= \underbrace{\mu\sum_i \omega_i z_i}_{\text{income share of constrained}} + \underbrace{(1-\mu)\sum_i \omega_i z_i}_{\text{income share of unconstrained}} \left(1-\frac{\lambda_i}{1+r}\right).
\end{aligned}
$$

The rest of the first column gives the path of *intertemporal* marginal propensity to consume out of an income shock:

$$
iMPC_t = (1-\mu)\sum_i \omega_i z_i \lambda_i^t \left(1-\frac{\lambda_i}{1+r}\right).
$$

The income shock is spent down at a rate λ_i.

Jacobian with respect to the interest rate. The sequence space Jacobian $\mathbf{M}_i^r$ describes the consumption response to a shock on r_t. It is characterized by two different elasticities: the elasticity of intertemporal substitution and the intertemporal marginal propensity to consume. The elasticity of intertemporal substitution determines to which extent households want to exploit differences in relative prices while the intertemporal marginal propensity to consume determines the speed at which additional incomes are consumed. The strength of that intertemporal substitution is determined by the EIS[6] of the unconstrained household:

$$
\text{EIS} = \frac{\rho(\bar{e})\beta}{\sigma}\lambda_i \left[\left(1-\frac{\lambda_i}{1+r}\right)\bar{e} + \frac{\mathbb{E}[e'|\bar{e}]}{1+r}\right] \frac{1+r}{\bar{e}\mathbb{E}[e'|\bar{e}]\Pi_{\bar{e}\bar{e}}},
$$

which is an increasing function of λ_i[7]. When r increases in the future, consuming today is relatively more expensive. Today's consumption falls while future's consumption increases. Households postpone part of their consumption to later periods by increasing their savings, and then consume a fraction $\lambda_i^t[1-\lambda_i/(1+r)]$ at period t of this additional income[8].

Interpretation of λ_i. The variable λ_i determines the contemporaneous MPC, the persistence of the iMPC, and the EIS of unconstrained households. The MPC is a decreasing function of

[5] The classic MPC in the literature.

[6] Notice that, in the absence of taste for wealth and idiosyncratic shocks, the formula simplifies to $1/\sigma$.

[7] Indeed, recall that λ_i is between 0 and 1. The negative second-order term of this polynomial will thus be neglectible and dominated by the positive linear term.

[8] Since there is no wealth in our model, interest rate shocks have no initial income effects and an unexpected change in r_0 has no impact on consumption. The first column of $\mathbf{M}_i^r$ is thus zero.

λ_i while the EIS is an increasing function of λ_i. It is thus key in understanding the behavior of households as in it pins down the degree to which households are forward-looking.

When $\lambda_i = 1$, unconstrained households behave as forward-looking, permanent-income households in a riskless world. Their consumption response to an income shock is perfectly smoothed and they consume at every period a fraction $r/(1+r)$ of that income shock. In that case, the indirect effect is low as the consumption response of unconstrained households does not move much with transitory income shocks. Conversely, the consumption of unconstrained households is very sensitive to changes in the real interest rate as it controls the relative prices of consumption at different time periods. As households put a large weight on future consumption, their consumption response is very sensitive to a change in relative prices of future consumption. As a result, the direct effect of a monetary shock is high.

When $\lambda_i < 1$, households are less forward-looking. There is now an extra discounting of future consumption flows. Their contemporaneous MPC is higher than future iMPC. In that case, the indirect effect is high as the consumption response of unconstrained households largely moves with a contemporaneous income shock. Conversely, the consumption of the unconstrained household is less sensitive to changes in the real interest rate as the EIS decreases.

1.3.2 The aggregate effect of a monetary policy shock

Combining Proposition 1 and the analytical solutions for our two sequence-space Jacobians, we compute the aggregate effect of a monetary policy shock $r_0 < r^{ss}$ with persistence ρ. We also compute the direct and the indirect effects.

Proposition 4. *The aggregate effect of a monetary policy shock r_0 is given by:*

$$dY_0 = -\beta\rho(\bar{e})\frac{1}{\sigma}\frac{\rho}{1-\rho}dr_0. \tag{2}$$

The direct effect of a monetary policy shock is given by:

$$\textit{Direct effect}_0 = -(1-\mu)\rho(\bar{e})\frac{1}{1+r}\frac{1}{\sigma}\sum_i \omega_i z_i \frac{\rho\lambda_i\beta}{1-\rho\lambda_i\beta}dr_0.$$

The indirect effect of a monetary policy shock is given by:

$$\textit{Indirect effect}_0 = dN_0 - (1-\mu)\frac{1}{1+r}(1-\rho)\sum_i \omega_i z_i \frac{\lambda_i}{1-\beta\lambda_i\rho}dN_0.$$

The direct effect is an increasing function of λ_i while the indirect effect is an decreasing function of λ_i.

Proof. Appendix A4 □

The structure of preferences and the level of ex-ante heterogeneity matter for monetary policy as they jointly determine the value of λ_i and the equilibrium r. At the aggregate level, λ_i determines the relative weight of the direct and indirect effects of a monetary shock on output. The larger λ_i, the higher the direct effect and the lower the indirect effect. This is consistent with the fact that the static MPC is a decreasing function of λ_i while the EIS is an increasing function of λ_i. In the next section, we are going to study what happens when ex-ante heterogeneity rises (1) in the absence of a non-homothetic taste for wealth ($\gamma = 0$) and (2) in the presence of a non-homothetic taste for wealth ($\gamma > 0$).

1.3.3 Benchmark: monetary policy without a taste for wealth

We first study this model in the benchmark case without a taste for wealth by setting γ to 0.

Proposition 5. *In the HANK model with zero-liquidity, when $\gamma = 0$ (no preference for wealth), the Jacobians $\boldsymbol{M}_i^r$ and $\boldsymbol{M}_i^n$ do not depend on z:*

$$\forall i \in \{l, h\}, x \in \{r, n\}, \quad \Delta \mathbf{M}_i^x(z') = 0 \textit{ and } \mathbf{M}_l^x = \mathbf{M}_h^x.$$

1. *There is neither a composition effect nor a behavior effect.*
2. *A change in permanent labor income inequality does not affect the magnitude of a monetary policy shock.*

Proof. Appendix C □

Without a taste for wealth, the slope of the steady-state savings function λ_i does not depend on the level of permanent income. $\forall i,\ \lambda_i \equiv \lambda$. Thus, the share of constrained households across types is the same, and the sequence-space Jacobians per unit of permanent income are equal across types. The distribution of permanent income is neutral on the transmission of monetary policy.

Indeed, without a non-homothetic taste for wealth, differences in permanent productivity scale linearly consumption and savings functions across permanent types (Straub 2019). As the slope of the saving function of the highest idiosyncratic type evaluated at the steady state is constant across permanent types, the sequence-space Jacobians per unit of permanent income are equal across types. As a result, the total, direct, and indirect effects do not depend on the distribution of permanent productivity. A change in the distribution of permanent income thus does not affect the output response to a monetary policy shock, nor the transmission channel of monetary policy.

1.3.4 Accounting for the non-homotheticity in saving behavior

We now study the impact of an increase in permanent income inequality in the zero-liquidity model under non-homothetic preferences. In the presence of a taste for wealth, all low permanent income types are constrained, $\lambda_l = 0$ and $\lambda \equiv \lambda_h$[9]. A rise in permanent labor income inequality changes the transmission channels of monetary policy, leaving the aggregate effect constant.

Permanent income and behavior effect. Under non-homothetic preferences, λ decreases with permanent income because r is itself a decreasing function of permanent income. Thus, an increase in permanent labor income inequality increases the MPC, decreases the persistence of the iMPC, and decreases the EIS. In Proposition 2, this change in λ and r determine the behavior effects $\Delta \mathbf{M}_h$ which is the change in the coefficients of our sequence-space Jacobians *per unit of permanent income.*

When permanent labor income increases under non-homothetic preferences, the Euler equation is more "discounted" in the terms of McKay et al. (2017) and Michaillat & Saez (2021). Indeed, as the high-productivity households become richer, their consumption increases, decreasing their marginal utility from consumption. For their Euler equation to hold at the steady state, the real interest rate has to fall, since

$$1 + r = \frac{1}{\beta\rho(\bar{e})}\left[1 - \frac{\gamma\zeta^{-\Sigma}}{(z_h\bar{e})^{-\sigma}}\right], \quad \frac{dr}{dz_h} < 0.$$

[9]The Euler equation now depends on z_i and the real interest rate will adjust so that the Euler equation will hold with equality only for the high permanent income type with the highest idiosyncratic type. See Appendix B.2 for details.

Outside of the steady state, unconstrained households now put a lower weight $\beta(1+r_{t+1}) < 1$ on future consumption since the Euler equation writes

$$c_t^{-\sigma} = (1 + r_{t+1})\beta\mathbb{E}[c_{t+1}^{-\sigma}] + \gamma(a_{t+1} + \zeta)^{-\Sigma}.$$

As a result, a change in the path of real interest rates, i.e. a change in relative prices of consuming at different time periods has a lower effect on consumption today (as future consumptions are more discounted). Unconstrained households would like to hold wealth relatively more for a *present motive* due to the non-homothetic nature of the taste for wealth rather than for an intertemporal substitution motive. At the micro level, unconstrained households become less sensitive to variations in the real interest rate, their EIS falls. At the same time, they frontload their consumption reaction to an income shock, their MPC rises. At the macro level, the direct effect is weakened while the indirect effect is strengthened[10].

Composition effect. A change in permanent income also redistributes a higher share of income towards households with a lower MPC and a higher EIS, since all the constrained households have a MPC of 1 and an EIS of 0. Formally, it changes the weights on each sequence-space Jacobian $\Delta\mathbf{M}_i$, which increases the direct effect and decreases the indirect effect.

To sum up, an increase in permaneent labor income inequality entails both a composition and a behavior effect, and both those effects impact the transmission channels of monetary policy:

$$\frac{d\text{Indirect effect}_0}{dz_h} = \underbrace{\text{composition effect}_0}_{<0} + \underbrace{\text{behavior effect}_0}_{>0},$$
$$\frac{d|\text{Direct effect}_0|}{dz_h} = \underbrace{\text{composition effect}_0}_{>0} + \underbrace{\text{behavior effect}_0}_{<0}.$$

Furthermore, with zero liquidity, the change in the permanent labor income distribution has no effect on the incidence of monetary policy on output[11]. The "as-if" result of Werning (2015) holds in this framework. The change in the income share of constrained and unconstrained households generates no aggregate amplification since, in the absence of government bonds, the elasticity of the net income of hand-to-mouth households to aggregate income is one (Bilbiie 2008). In a simulation exercise done in Section 1.4, the composition effect dominates

[10] We prove this formally in Appendix XXX

[11] Note that Equation 2 depends neither on r nor on λ.

the behavior effect. The rise in permanent labor income inequality hence leads to a rise in the indirect effect and a fall in the direct effect. The results are displayed in Table 2.

1.4 HANK with positive liquidity

We now relax the zero-liquidity assumption and allow for a realistic level of liquid wealth. The distribution of wealth is no longer degenerate, with a mass of agents bunched at zero wealth. An increase in permanent income inequality changes the distribution of wealth, which in turn impacts the aggregate consumption response to a monetary policy shock.

1.4.1 Calibration

For the distribution of permanent labor income, we set ω to 0.1 and chose z_h so as to match the labor income share held by the top 10% in 1989 and in 2019. We follow Straub (2019) to calibrate the CRRA parameter on wealth in order to match an elasticity of consumption to permanent income of 0.7. The last two remaining parameters γ, the strength of the taste for wealth, and ζ, the Stone-Geary parameter, are calibrated with 2019 level of permanent labor income inequality so as to match a static aggregate MPC of 0.51 and a one-year intertemporal MPC of 0.18 as in Auclert et al. (2024). When we decrease the level of permanent labor income inequality to its 1989 level, we keep the rest of the calibration constant. Table 1 summarizes our calibration.

1.4.2 Results

The introduction of positive liquidity results in a non-degenerate distribution of wealth that can endogenously vary with the distribution of permanent labor income. This shift in the distribution of wealth alters the results obtained in the zero-liquidity case. The rise in permanent labor income inequality increases the size of the indirect effect and decreases the size of the direct effect (see table 2). The increase in the indirect effect outweighs the reduction in the direct effect, leading to a larger overall impact of a monetary shock on output following the rise in permanent labor income inequality.

Indirect effect. The rise in the indirect channel comes from an increase in the aggregate MPC (see table 3) which ultimately stems from three effects: the change in policy functions, the change in the wealth distribution, and the composition effect.

Table 1: Calibration of the one-asset HANK model

Parameter	Description	Value	Source
Household			
β	Discount factor		To match $r = 5\%$
σ	CRRA on consumption	1	Auclert & Rognlie (2020)
γ	Strength of the taste for wealth	0.03	Internally calibrated
ζ	Stone-Geary shifter	0.4	Internally calibrated
Σ	CRRA on wealth	0.7	Straub (2019)
Labor income			
ρ_e	Autocorrelation productivity shocks	0.92	Straub (2019)
σ_e	s.d. productivity shocks	0.2	Straub (2019)
ω	High permanent labor income type	0.1	Internally calibrated
z_h	High productivity type	$\{2.8, 3.4\}$	Piketty et al. (2018)
Government			
r_{ss}	Steady state interest rate	5%	
ρ_r	Autocorrelation of monetary shock	0.15	Kaplan et al. (2018)
σ_r	s.d. of monetary policy shock	0.01	Kaplan et al. (2018)
B	Government debt	0.23	Auclert & Rognlie (2020)

First, the rise in permanent labor income inequality leads to a change in policy functions. For a given level of wealth, high-permanent income households have higher MPC after the rise in their permanent income (see Section 1.3) which tends to increase the aggregate MPC. This is shown in the left panel of Figure 2.

Second, the rise in permanent labor income inequality leads to a change in the wealth distribution. In particular, we observe a significant rise in the share of hand-to-mouth households. Indeed, when permanent labor income inequality rises, high-permanent income households put a relatively higher weight on the taste for wealth, as wealth is a luxury good. They want to increase savings everything else equal. For the asset market to clear, the real interest rate has to fall. However, at the bottom of the income distribution, households put a relatively lower weight on the taste for wealth. Their savings decision is hence driven mostly by the precautionary and intertemporal substitution motives so that the decrease in the real interest

Table 2: Change in total output elasticity and partial effect

	Partial effect			Total effect
	r	N	Tax	Y
Zero liquidity	0.28	-0.27	-0.01	0
Positive liquidity	-1.91	2.74	-0.83	7.75

Note: The three first columns of this table describe the change in the percentage of total output explained by the partial effect in an economy with a high and a low level of permanent labor income inequality in the first period. For example, the first number means that in the zero liquidity model, the share of output change explained by the direct effect 'r' decreased by 0.27%. The last column shows the percentage change in the elasticity of total output to a monetary policy shock.

rate pushes them to dissave. As a result, the share of hand-to-mouth households increases, as shown in the left panel of 2. This wealth distribution effect tends to increase the aggregate MPC.

Table 3: Change in the MPC

	Low type	High type	Aggregate
Low inequality	0.38	0.3	0.37
High inequality	0.41	0.3	0.4

Lastly, the composition effect tends to dampen the rise in the aggregate MPC. Indeed, as the average MPC is higher among low permanent-income households than among high permanent-income households, the rise in permanent labor income inequality decreases the aggregate MPC. Quantitatively, the sum of the first two effects largely dominates the last one leaving the aggregate MPC higher after the rise in permanent labor income inequality.

Direct effect. The change in the direct effect occurs in the opposite direction of the change in the indirect effect. First, the change in the policy function reduces the EIS, as it decreases with rising permanent income (see Section 1.3), leading to a lower aggregate EIS. Second, the change in the wealth distribution further diminishes the direct effect, as a larger proportion of households are now hand-to-mouth and thus unresponsive to interest rate changes. Finally, the composition effect reallocates income towards households with a higher EIS (see the first two columns of Table 4), increasing the aggregate EIS. However, quantitatively, the combined

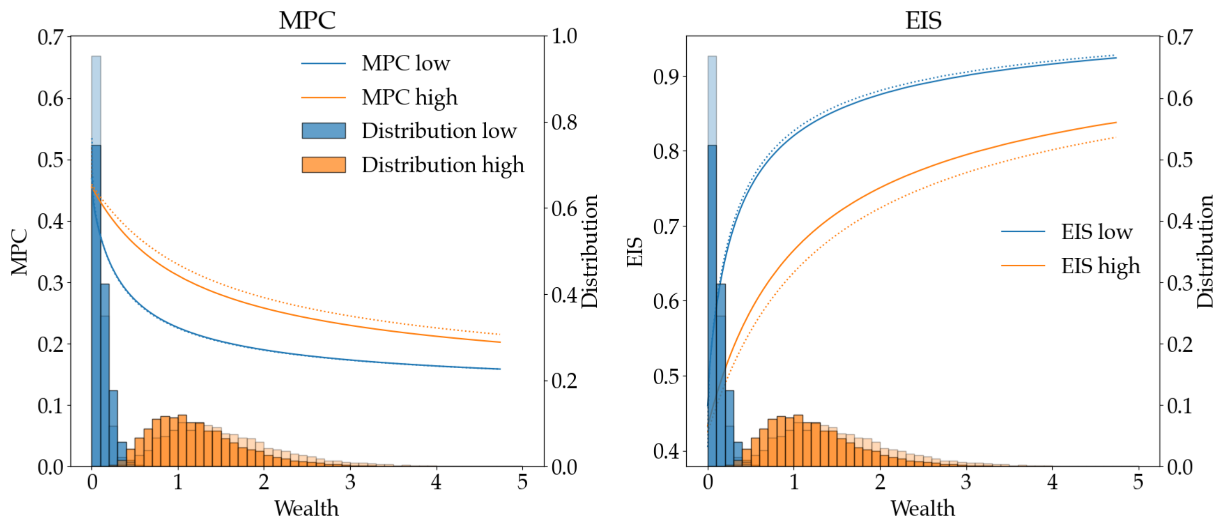

Figure 2: MPC and EIS along the wealth distribution

Note: This figure plots the marginal propensity to consume out of a one-time income shock (on the left) and the elasticity of intertemporal substitution (on the right) for different levels of wealth (x-axis) and for different levels of permanent income (in orange, the permanent labor income of the top 10% in 1989, in dotted orange the permanent labor income of the top 10% in 2019; in blue, the permanent labor income of the bottom 90% in 1989, in dotted blue the permanent labor income of the bottom 90% in 2019. We also plot the wealth distributions of the two permanent income types normalized (in blue, the low-income type, and in orange, the high-income type) before and after the rise in permanent labor income inequality (in dark color before and transparent after the rise).

impact of the first two effects outweighs the third, resulting in a lower aggregate EIS following the rise in permanent labor income inequality (see the last column of Table 4).

Amplification. The aggregate neutrality of the permanent labor income distribution no longer holds with a realistic level of wealth. The rise in the share of hand-to-mouth households amplifies the output response to a monetary shock. Indeed, the elasticity of hand-to-mouth net income to aggregate income is greater than one. In the case of a fall in the real interest rate, the net income of hand-to-mouth households increases both from the rise in labor demand and from the fall in the labor tax rate (Bilbiie 2008). Moreover, even if the change in policy functions and distributions might be small quantitatively, the overall impact on the output response to a monetary shock remains economically significant since those composition, behavior, and wealth effects are amplified by the change in labor income (see Proposition 2).

Table 4: Change in the EIS

	Low type	High type	Aggregate
Low inequality	0.59	0.68	0.6
High inequality	0.55	0.68	0.57

1.4.3 Impact of an increase in the variance of idiosyncratic shocks

Does the nature of the rise in labor income inequality matter for monetary policy? In other words, what would have happened if the rise in labor income inequality were to be driven by an increase in the variance of shocks happening over the lifetime. The results are displayed in Figure 5 in the appendix.

In the zero-liquidity case, an increase in the variance of shocks decreases λ and raises $\rho(\bar{e})$. The fall in λ, through a rise in the aggregate MPC, increases the indirect response. At the same time, the rise in $\rho(\bar{e})$ magnifies the intertemporal substitution effect. As high idiosyncratic type households earn more, a change in the relative price of consumption has a higher effect on their aggregate consumption. Combining the rise in indirect and direct effects, an increase in the variance of shocks unambiguosly increase the effect of monetary policy in the zero-liquidity case.

This property no longer holds when households have access to a sufficiently-large stock of government bonds. Indeed, there is now an additional effect. When the variance of shocks increases, more households end up at or close to the borrowing constraint. The EIS of those households is low and so the direct effect is weakened. On top of that, there is an aggregate dampening of a monetary shock as the equilibrium r decreases when labor income inequality increases[12]. The lower interest rate means that the equilibrium tax rate is lower and the elasticity of the disposable income of hand-to-mouth households to aggregate income decreases (but remains above 1). There is less amplification and a monetary shock has a lower aggregate effect after the rise in the variance of idiosyncratic shocks.

We see that the predictions of the effect of a rise in the variance in initial outcomes compared to the effect of a rise in the variance in idiosyncratic shocks are largely reversed. The nature

[12] As households now want a higher buffer stock of savings, everything else equal.

of the rise in labor income inequality does matter to understand the change in its aggregate effect and its composition.

2 A quantitative two-asset HANK model

Our previous model featured only one endogenous variable, N_t. It also abstracted from a realistic supply side, investment and inflation, in order to represent the entire path of the economy through an intertemporal-Keynesian cross. In this section, we extend our previous analysis to a quantitative environment that features a two-asset choice on the household side and includes the previously omitted elements.

2.1 Environment

Households. The problem of a household of type i in labor union k[13] is given by:

$$V_{i,t}(\ell_t, a_t, e_t) = \max_{c_t} \left\{ \frac{c_t^{1-\sigma} - 1}{1-\sigma} - \varphi \frac{n_{kt}^{1+\nu}}{1+\nu} + \gamma \frac{(a_t + \ell_t + \zeta)^{1-\Sigma} - 1}{1-\Sigma} + \beta \mathbb{E}_t[V_{i,t+1}(a_{t+1}, \ell_{t+1}, e_{t+1}) | e_t] \right\},$$

subject to the law of motion,

$$\begin{aligned} c_t + \ell_{t+1} &= \lambda_t (z_i e_t w_{kt} n_{kt})^{1-\tau} + (1 + r_t^{\ell})\ell_t + d_t, \\ a_{t+1} &= (1 + r_t^a) a_t - d_t(a_t). \end{aligned}$$

We follow Auclert et al. (2020) and assume that flows from and towards the illiquid are determined exogenously by the following rule

$$d_t(a_t) = \frac{r_{ss}^a}{1 + r_{ss}^a} (1 + r_t^a) a_t + \Omega \left((1 + r_t^a) a_t - (1 + r_{ss}^a) \bar{a}_i \right)$$

where $\bar{a}_i$ is a target level of illiquid wealth determined exogenously, and Ω is a number close to zero. This rule implies that, at the steady state, households receive a fixed income $r_{ss}^a \bar{a}_i$ from their illiquid account and that the total supply of illiquid assets is $\sum_i \omega_i \bar{a}_i$. In the transition, they will transfer the excess or missing returns toward the liquid account. The evolution of the idiosyncratic productivity shocks follows a log AR(1) process given by

$$\log(e_t) = \rho \log(e_{t-1}) + \varepsilon_t^e,$$

[13] See next section.

with $\varepsilon_t^e \sim \mathcal{N}(0, \sigma_e^2)$. Finally, notice that the number of hours supplied is not a control variable as it is chosen by the labor union. This household block gives rise to the aggregate consumption and the aggregate asset supply functions

$$L_t = \mathcal{L}_t(\{r_s, N_s, \lambda_s\}_{s\geq t}, \mu_t),$$
$$A_t = \mathcal{A}_t(\{r_s, N_s, \lambda_s\}_{s\geq t}, \mu_t),$$
$$C_t = \mathcal{C}_t(\{r_s, N_s, \lambda_s\}_{s\geq t}, \mu_t),$$

where μ_t is the distribution of households over idiosyncratic states, asset positions, and permanent types.

Unions. Every worker belongs to a union k. Each union k aggregates efficient units of work into a union-specific task N_{kt} and sells those union-specific tasks at price W_{kt} to a competitive labor packer with an elasticity of substitution of ε between union-specific tasks. The problem of the union is specified in the Appendix. Solving the union problem yields the following non-linear New-Keynesian Phillips curve on wage inflation:

$$\pi_t^w = \kappa \left(\varphi \left(N_t\right)^{\nu} - \frac{1}{\mu} \left(1 - \lambda_t\right) w_t \left(C_t\right)^{-\sigma} \right) + \beta \pi_{t+1}^w$$

Central bank. The central bank follows a Taylor rule

$$i_t = r + \phi_\pi \pi_t + \varepsilon_t^r,$$

where r is the steady-state interest rate. The innovations to the nominal interest rate set by the central bank follow an AR(1) process with persistence ρ_r. The *ex-ante* real interest rate is given by the Fisher equation

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}.$$

Firms. The firm block of our model follows Auclert & Rognlie (2020). The supply side is composed of two sectors. A final goods producer that uses a basket of intermediate inputs $x_{i,t}$ with an elasticity of substitution $\mu_p/(\mu_p - 1)$, and a continuum of intermediate firms in imperfect competition, which produces the intermediate goods using capital and labor with a Cobb-Douglas technology production function. Intermediate goods are produced by a mass one of identical monopolistically competitive firms, whose shares v_t are traded, with price p_t and dividends d_t at time t, and owned by households. Intermediate firms own the stock of physical capital, make investment decisions subject to convex adjustment, and make pricing decisions subject to Rotemberg adjustment costs. The intermediate's firm problem can be found in Appendix F.1 This setup yields a standard non-linear New-Keynesian Phillips curve on final-goods price inflation[14]:

$$\pi_t (1+\pi_t) = \kappa^p (\mu^p \cdot mc_t - 1) + \frac{1}{1+r_t}\pi_{t+1}(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}.$$

where the marginal cost is given by, $mc_t = \frac{1}{1-\alpha} w_t \frac{N_t}{Y_t}$, with w_t the real wage, κ^p the parameter governing the Rotemberg cost, and μ^p the markup of intermediate firms. The investment decision is governed by the usual marginal Tobin's Q equations

$$Q_t = 1 + \frac{1}{\delta \epsilon_I}\left(\frac{K_t}{K_{t-1}} - 1\right),$$

$$(1+r_{t-1}) Q_{t-1} = mc_t \cdot F_K(K_t, N_t) - \left(\frac{K_{t+1}}{K_t} - (1-\delta)\right) - \varphi\left(\frac{K_{t+1}}{K_t}\right) + \frac{K_t}{K_t} Q_t.$$

Finally, the price of the firm p_t is given by the non-arbitrage condition between owning equity in the firm and government bonds:

$$p_t = \frac{d_{t+1} + p_{t+1}}{r_{t+1}}.$$

Financial intermediary. There is a financial intermediary that receives liquid and illiquid deposits from households and invests them in equity and government bonds. It performs this liquidity transformation at a fixed cost ξ so that $r_t = r_t^a = r_t^b - \xi$. We assume that the financial intermediary is in perfect competition so that it makes no profit in equilibrium.

The flow-of-fund constraint at the beginning of period t states that the value of liabilities must be equal to the liquidation value of the intermediaries portfolio:

$$(1+r_t^a) A_t + \left(1 + r_t^l\right) L_t = (1+r_t) B_t + (p_t + d_t) v_t - \xi L_t.$$

At the end of period t, new investments in bonds and shares should be equal to intermediary liabilities (or aggregate savings):

$$p_t v_{t+1} + B_{t+1} = A_{t+1} + L_{t+1}$$

Finally, returns on the illiquid assets are subject to capital gains following a one-time unexpected shock. The return on illiquid assets in period 0 following an unexpected shock is given by

$$r_0^a = \frac{p_{ss}}{B + p_{ss}}\frac{d_1 + p_1}{p_0} + \frac{B_{ss}}{B + p_{ss}} r_0.$$

[14] Appendix G.1 in Auclert et al. (2024)

The firm term of this expression captures the fact that following an unexpected shock, the price of the firm might jump (or fall) unexpectedly, increasing (or decreasing) the real return on equity compared to what households expected to receive the period before the shock, at the steady state. Due to perfect foresight, the usual non-arbitrage condition holds for all the remaining transition periods.

Government. The government imposes a progressive tax on labor income, determined by the HSV coefficient τ and the rate λ_t. The budget constraint is

$$N_t w_t = \lambda_t \sum_i \omega_i \int (w_t e_t s_i N_t)^{1-\tau} d\mu_{i,t} + r_t B.$$

Market clearing. Market clearing implies that

1. Asset market: $A_t = p_t + B$;
2. Labor market: $N_t^s = N_t^d$;
3. Goods market: $Y_t = C_t + I_t + G + \xi L_t + \varphi\left(\frac{K_{t+1}}{K_t}\right) + \kappa_p\left(\frac{P_{t+1}}{P_t} Y_t\right)$.

2.2 Calibration

Calibration of permanent types

Our quantitative model has four types of households, each belonging to a group of the distribution of permanent income: the **b**ottom **50%**, the **n**ext **40%**, the **n**ext **9%** and the **t**op **1%** of the distribution of permanent income. We calibrate the corresponding z_i to match the distribution of labor income described in Piketty et al. (2018). We then associate each of those groups to a target of illiquid asset $\bar{a}_i$, as in Auclert et al. (2020). We set those targets to match the unconditional distribution of illiquid assets in the Survey of Consumer Finance, and to match a share of liquid assets to output of 0.23, as in Kaplan et al. (2018), Auclert & Rognlie (2020). The final distribution is described in table XXX. In our main computational exercise, we compute the impact of a monetary policy shock in an economy where the $(s_{b5}, s_{n40}, s_{n9}, s_{t1})$ and $(a_{b5}, a_{n40}, a_{n9}, a_{t1})$ are calibrated to match the distribution of income in 2019 (high inequality) and 1989 (low inequality).

Calibration of the taste for wealth

Table 5: Calibration of the quantitative model

Parameters	**Description**	**Value**	**Source**
Preferences			
β	Discount factor	0.85	Internally calibrated
σ	CRRA coefficient on consumption	1	Auclert et al. (2024)
γ	Strength of the taste for wealth	0.7	Internally calibrated
Σ	CRRA coefficient on wealth	0.7	Straub (2019)
ζ	Stone-Geary parameter	0.25	Internally calibrated
Productivity			
σ_z	Variance of idiosyncratic productivity shocks	0.2	Straub (2019)
ρ_z	Autocorrelation of shocks	0.91	Straub (2019)
Union			
κ_w	Slope of NKPC	0.03	Auclert et al. (2024)
μ_w	Markup of the union	1.01	Auclert et al. (2024)
Firm			
α	Cobb-Douglas coefficient for capital	1/3	
δ	Depreciation rate	8%	Auclert et al. (2024)
μ_p	Markup of the firms	1.01	Auclert et al. (2024)
ε_I	Investment cost parameter	4	Auclert et al. (2024)
κ^p	Rotemberg cost parameter	0.23	Auclert et al. (2024)
Portfolio choice			
Ω	Flows from illiquid account	0.005	Auclert et al. (2020)
ζ	Illiquidity premium	2%	Auclert et al. (2024)
r_{ss}	Steady-state interest rate	5%	
Government			
G	Government spendings	0.2	
τ	Progressiveness of labor tax	0.181	Auclert et al. (2024)
B	Public debt	0.7	Auclert et al. (2024)
Monetary policy			
r_{ss}	Steady state interest rate	5%	Auclert et al. (2024)
ϕ_π	1.5		
ρ_r	0.15		

We calibrate internally only two parameters: the strength of the taste for wealth γ and the Stone-Geary shifter ζ. We set those parameters to match the distribution of wealth in our model in 2019, given a distribution of permanent labor income $(s_{b5}, s_{n40}, s_{n9}, s_{t1})$ and a distribution of illiquid wealth $(a_{b5}, a_{n40}, a_{n9}, a_{t1})$. Table **??** describes the total distribution of wealth in our economy[15]. Our model also does a fairly good job at reproducing the distribution of wealth in 1989, which we do not target in the calibration of the taste for wealth (overestimating the wealth share of the middle 40% and underestimating the top 10% wealth share).

Notice that, when calibrating, we do not target the path of iMPC, only the wealth distribution. However, we do get quite close on the aggregate MPC from Fagereng et al. (2021), but we undershoot the iMPC one year after the income shock estimated in Auclert et al. (2024) (from 0.16 to 0.18 depending on the data source).

Due to the two-asset structure, the share of hand-to-mouth aggregates both poor hand-to-mouth households that have no liquid and illiquid wealth and wealthy hand-to-mouth households that have no liquid wealth but have positive illiquid wealth. The later are reluctant to dissave their illiquid wealth. As a result, they feature high MPC out of any labor income shock even if they have positive weealth.

Table 6: Distribution of wealth in the model and the data

	Bottom 50%	Middle 40%	Top 10%	Top 1%
Data PSZ 2019	0	28	71	34
Model 2019	1	28	69	38
Data PSE 1989	1	33	64	28
Model 1989	3	37	59	29

2.3 Results

We now investigate the impact of rising permanent labor income inequality in this quantitative framework. In our main experiment, we study the impact of an expansionary monetary

[15] We use the extended series available on `https://gabriel-zucman.eu/`.

policy shock in two different economies: one where the distribution of permanent labor income and illiquid wealth is calibrated on the U.S. economy in 1989, and one where those distributions are calibrated on 2019[16].

Table 7: Decomposition of the effect of a monetary policy shock

	Low inequality	High inequality	Change
Elasticity of Y	-2.13	-2.4	12.4 %
Elasticity of C	-1.46	-1.92	31.4%
Elasticity of I	-3.36	-3.19	-4.86%
Elasticity of C, part. eq.	-0.5	-0.37	-25.7 %
Component of % change of C due to			
Direct effect of r^b	36	20	-16 p.p.
Indirect effect of N	28	42	14 p.p.
Indirect effect of taxes	33	36	3p.p.
Indirect effect of p	3	2	-1p.p.

Note: Average response over the first year. The first column reports the number in the economy with low permanent labor income inequality, calibrated on 1989. The second column reports the number in the economy with high permanent labor income inequality, calibrated on 2019. The last column reports the % change of the elasticities, and the p.p. for the partial effects.

Table 7 reports the main results of this quantitative exercise. We find that the increase in permanent labor income inequality increases the output elasticity to a monetary policy shock by 12.4%, a result in line with our findings in the previous section. This increase is mostly driven by a higher sensitivity of consumption to changes in labor income, as reported by the second part of Table 7, and attenuated by a lower sensitivity of consumption to changes in the interest rate. Indeed, the partial elasticity of consumption decreases by 25% in our model, while the share of total change in consumption, explained by the indirect effects, increases significantly.

[16] We run the same exercise as in Section 2.3, but keep the distribution of illiquid wealth constant. Looking at Table 9 and 10, we get similar results as before.

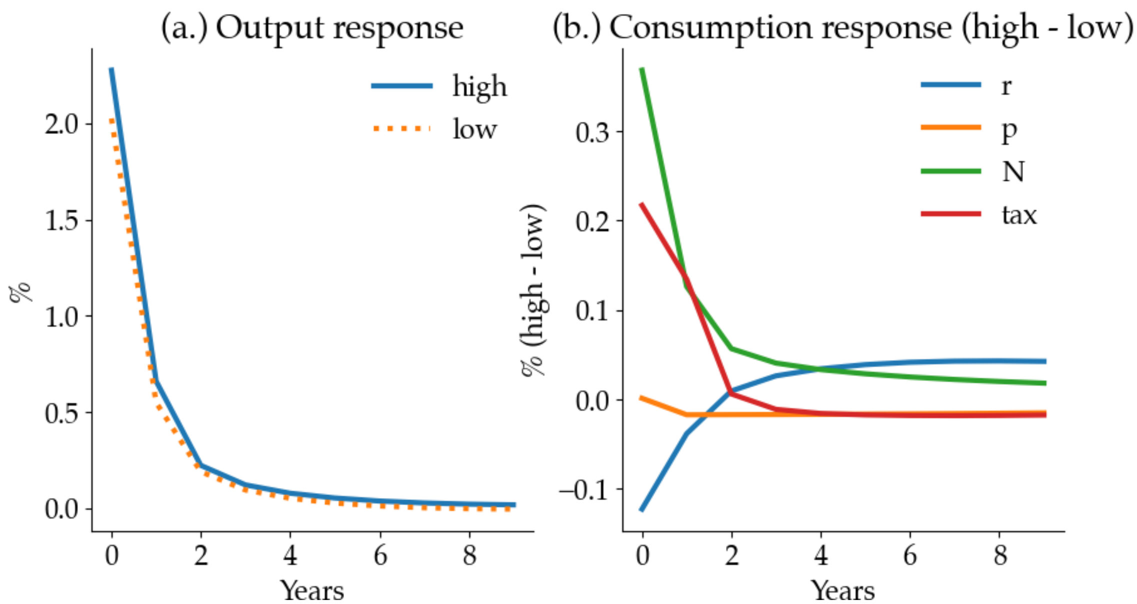

Figure 3: Effect on output of a monetary shock

Note: The left figure plots the output response following a monetary shock. The right figure plots the difference in the decomposition between the high-inequality economy minus the low-inequality economy.

What are the implications of an increase in permanent labor income inequality on the transmission of monetary policy with a realistic wealth distribution?

Our results remain consistent with the findings from Section 1.4. An increase in labor permanent income inequality increases the effect of a monetary shock. Looking at the decomposition in Figure 3 (b.), the contribution of indirect effects (from taxes and the labor demand) increases while the contribution of the direct effect is slightly lower. Consistently with Section 1.4, the rise in indirect effects comes from the increase in the income share going to hand-to-mouth households as the wealth-distribution effect dominates the composition effect. The endogeneous rise in the mass of hand-to-mouth among low-permanent income types dominates the fall in the income share going to low-permanent income households. The wealth distribution effect is reinforced by the policy function effect. Indeed, the rise in permanent income increases the marginal propensity to consume of high-permanent income households. Not surprisingly, in Figure 4 (a.), we see that the aggregate MPC goes up while the subsequent iMPCs decrease.

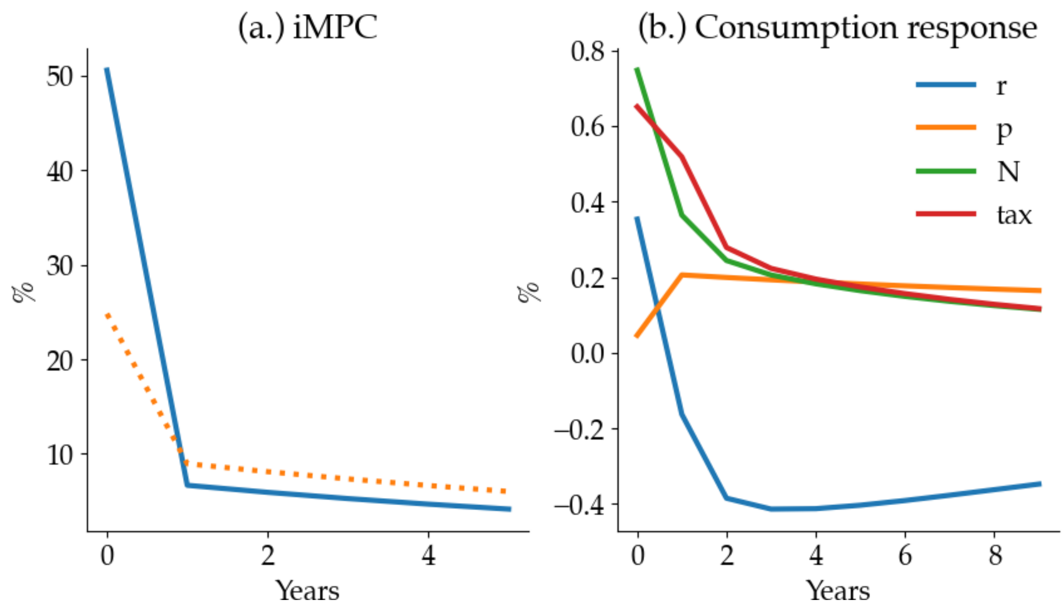

Figure 4: iMPC and decomposition of a monetary shock

Note: The left figure plots the iMPC. The right figure plots the decomposition of a monetary policy shock in the high-inequality economy.

At the same time, the rise in permanent labor income inequality tends to dampen the aggregate elasticity of intertemporal substitution. Similarly as in Section 1.4, the composition effect tends to raise the aggregate EIS by giving a higher income share to households that have *on average* a higher EIS. However, the composition effect is largely dominated by the sum of the policy function effect and the wealth distribution effect.

Conclusion

In this paper, we have shown that the rise in permanent labor income inequality changes the composition of a monetary shock by decreasing the share of direct and increasing the share of indirect effects. Decomposing the effect of the rise in permanent labor income inequality on monetary policy, we show that our main result is attributable to the combination of the policy function effect and the wealth distribution effect which outweigh the composition effect. The change in the permanent labor income distribution also generates amplification of monetary policy through the change in the income share going to hand-to-mouth households.

References

Auclert, A. (2019), 'Monetary policy and the redistribution channel', *American Economic Review* **109**(6), 2333–2367.

Auclert, A., Bardóczy, B., Rognlie, M. & Straub, L. (2021), 'Using the sequence-space jacobian to solve and estimate heterogeneous-agent models', *Econometrica* **89**(5), 2375–2408.

Auclert, A. & Rognlie, M. (2020), 'Inequality and Aggregate Demand', p. 82.

Auclert, A., Rognlie, M. & Straub, L. (2020), 'Micro jumps, macro humps: Monetary policy and business cycles in an estimated hank model', *Working paper* .

Auclert, A., Rognlie, M. & Straub, L. (2024), 'The Intertemporal Keynesian Cross', *Working paper* .

Bilbiie, F. O. (2008), 'Limited asset markets participation, monetary policy and (inverted) aggregate demand logic', *Journal of economic theory* **140**(1), 162–196.

Bloom, N., Guvenen, F., Pistaferri, L., Sabelhaus, J., Salgado, S. & Song, J. (2017), The great micro moderation, *in* 'Working Paper'.

Braxton, J. C., Herkenhoff, K. F., Rothbaum, J. L. & Schmidt, L. (2021), Changing income risk across the us skill distribution: Evidence from a generalized kalman filter, Technical report, National Bureau of Economic Research.

Carroll, C. D. (1998), 'Why do the rich save so much?'.

Coibion, O., Gorodnichenko, Y., Kueng, L. & Silvia, J. (2017), 'Innocent bystanders? monetary policy and inequality', *Journal of Monetary Economics* **88**, 70–89.

De Nardi, M., French, E. & Jones, J. B. (2016), 'Savings after retirement: A survey', *Annual Review of Economics* **8**, 177–204.

DeBacker, J. M., Heim, B. T., Vidangos, I. & Panousi, V. (2011), 'Rising inequality: Transitory or permanent? new evidence from a us panel of household income 1987-2006'.

Dynan, K. E., Skinner, J. & Zeldes, S. P. (2004), 'Do the rich save more?', *Journal of political economy* **112**(2), 397–444.

Fagereng, A., Holm, M. B. & Natvik, G. J. (2021), 'Mpc heterogeneity and household balance sheets', *American Economic Journal: Macroeconomics* **13**(4), 1–54.

Gaillard, A. & Wangner, P. (2021), 'Wealth, returns, and taxation: A tale of two dependencies', *Available at SSRN* **3966130**.

Guvenen, F., Kaplan, G., Song, J. & Weidner, J. (2022), 'Lifetime earnings in the united states over six decades', *American Economic Journal: Applied Economics* **14**(4), 446–479.

Holm, M. B., Paul, P. & Tischbirek, A. (2021), 'The transmission of monetary policy under the microscope', *Journal of Political Economy* **129**(10), 2861–2904.

Kaplan, G., Moll, B. & Violante, G. L. (2018), 'Monetary Policy According to HANK', *American Economic Review* **108**(3), 697–743.

Kaplan, G., Violante, G. L. & Weidner, J. (2014), The wealthy hand-to-mouth, Technical report, National Bureau of Economic Research.

Kumhof, M., Rancière, R. & Winant, P. (2015), 'Inequality, leverage, and crises', *American Economic Review* **105**(3), 1217–1245.

McKay, A., Nakamura, E. & Steinsson, J. (2017), 'The discounted euler equation: A note', *Economica* **84**(336), 820–831.

Melcangi, D. & Sterk, V. (2020), 'Stock market participation, inequality, and monetary policy', *FRB of New York Staff Report* (932).

Mian, A., Straub, L. & Sufi, A. (2020), The Saving Glut of the Rich, Technical Report w26941, National Bureau of Economic Research, Cambridge, MA.

Mian, A., Straub, L. & Sufi, A. (2021), 'Indebted Demand*', *The Quarterly Journal of Economics* **136**(4), 2243–2307.

Michaillat, P. & Saez, E. (2021), 'An economical business-cycle model', *Oxford Economic Papers* p. gpab021.

Michau, J.-B. (2018), 'Secular stagnation: Theory and remedies', *Journal of Economic Theory* **176**, 552–618.

Mumtaz, H. & Theophilopoulou, A. (2017), 'The impact of monetary policy on inequality in the uk. an empirical analysis', *European Economic Review* **98**, 410–423.

Piketty, T., Saez, E. & Zucman, G. (2018), 'Distributional National Accounts: Methods and Estimates for the United States*', *The Quarterly Journal of Economics* **133**(2), 553–609.

Saez, E. & Zucman, G. (2020), 'The rise of income and wealth inequality in america: Evidence from distributional macroeconomic accounts', *Journal of Economic Perspectives* **34**(4), 3–26.

Straub, L. (2019), 'Consumption, Savings, and the Distribution of Permamnent Income', p. 94.

Werning, I. (2015), Incomplete markets and aggregate demand, Technical report, National Bureau of Economic Research.

A Deriving the Intertemporal Keynesian Cross

A.1 Intertemporal Keynesian Cross following a monetary shock

In the goods space:

$$\mathbf{Y} = (1-\omega)\mathbf{C}_h(\{r_s, \tilde{N}_s\}) + \omega \mathbf{C}_l(\{r_s, \tilde{N}_s\})$$

Differentiating:

$$d\mathbf{Y} = (1-\omega)z_h \mathbf{M}_h^{\tilde{n}} d\tilde{\mathbf{N}} + (1-\omega)z_h \mathbf{M}_h^r d\mathbf{r} + z_l \omega \mathbf{M}_l^{\tilde{n}} d\tilde{\mathbf{N}} + \omega z_l \mathbf{M}_l^r d\mathbf{r}$$

In general equilibrium, $d\mathbf{Y} = d\mathbf{N} = d\mathbf{C}$, $d\boldsymbol{\tau} = Bd\mathbf{r} - rBd\mathbf{N}$, $\tau = rB$ and $d\tilde{\mathbf{N}} = (1-\tau)d\mathbf{N} - d\boldsymbol{\tau}$.

Hence, $d\tilde{\mathbf{N}} = (1-rB)d\mathbf{Y} - Bd\mathbf{r} + rBd\mathbf{Y} = d\mathbf{Y} - Bd\mathbf{r}$

We have that

$$d\mathbf{Y} = (1-\omega)z_h \mathbf{M}_h^{\tilde{n}}(d\mathbf{Y} - Bd\mathbf{r}) + (1-\omega)z_h \mathbf{M}_h^r d\mathbf{r} + z_l \omega \mathbf{M}_l^{\tilde{n}}(d\mathbf{Y} - Bd\mathbf{r}) + \omega z_l \mathbf{M}_l^r d\mathbf{r}$$

Solving for $d\mathbf{Y}$:

$$\left(\mathbf{I} - (1-\omega)z_h \mathbf{M}_h^{\tilde{n}} - z_l \omega \mathbf{M}_l^{\tilde{n}}\right) d\mathbf{Y} = \left((1-\omega)z_h \mathbf{M}_h^r + \omega z_l \mathbf{M}_l^r d\mathbf{r} - B(1-\omega)z_h \mathbf{M}_h^{\tilde{n}} - B z_l \omega \mathbf{M}_l^{\tilde{n}}\right) d\mathbf{r}$$

Using that $\forall x \in \{r, \tilde{n}\}, \quad \mathbf{M}^x = \omega z_l \mathbf{M}_l^x + (1-\omega)z_h \mathbf{M}_h^x$,

$$\left(\mathbf{I} - \mathbf{M}^{\tilde{n}}\right) d\mathbf{Y} = \left(\mathbf{M}^r - B\mathbf{M}^{\tilde{n}}\right) d\mathbf{r}$$

Mutiplying by $\mathbf{K} \equiv -\sum_{t=1}^{\infty}(1+r)^{-t}\mathbf{F}^t$ on both sides with $\mathbf{F}$ the forward matrix. Then inverting, we have that:

$$d\mathbf{Y} = \left(\mathbf{K}\left(\mathbf{I} - \mathbf{M}^{\tilde{n}}\right)\right)^{-1} \mathbf{K}\left(\mathbf{M}^r - B\mathbf{M}^{\tilde{n}}\right) d\mathbf{r}$$

A.2 Decomposition of the effect of a change in the permanent labor income distribution

In a economy characterized by a permanent labor income distribution, the effect of a monetary shock $d\mathbf{r}$ is given by

$$d\mathbf{Y}(\mathbf{z}) = ((1-\omega)z_h \mathbf{M}_h^r(\mathbf{z}) + \omega z_l \mathbf{M}_l^r(\mathbf{z}))d\mathbf{r} + ((1-\omega)z_h \mathbf{M}_h^n(\mathbf{z}) + \omega z_l \mathbf{M}_l^n(\mathbf{z}))d\tilde{\mathbf{N}}(\mathbf{z})$$

Taking the difference between the effect of the same monetary shock in two economies characterized by two different permanent income distribution $\mathbf{z}'$ and $\mathbf{z}$:

$$\begin{aligned} d\mathbf{Y}(\mathbf{z}') - d\mathbf{Y}(\mathbf{z}) =& ((1-\omega)z_h'\mathbf{M}_h^r(\mathbf{z}') + \omega z_l'\mathbf{M}_l^r(\mathbf{z}'))d\mathbf{r} + ((1-\omega)z_h'\mathbf{M}_h^n(\mathbf{z}') + \omega z_l'\mathbf{M}_l^n(\mathbf{z}'))d\tilde{\mathbf{N}}(\mathbf{z}') \\ & - ((1-\omega)z_h\mathbf{M}_h^r(\mathbf{z}) + \omega z_l\mathbf{M}_l^r(\mathbf{z}))d\mathbf{r} - ((1-\omega)z_h\mathbf{M}_h^n(\mathbf{z}) + \omega z_l\mathbf{M}_l^n(\mathbf{z}))d\tilde{\mathbf{N}}(\mathbf{z}) \end{aligned}$$

$$\begin{aligned} d\mathbf{Y}(\mathbf{z}') - d\mathbf{Y}(\mathbf{z}) =& ((1-\omega)(z_h'-z_h)\mathbf{M}_h^r(\mathbf{z}') + \omega(z_l'-z_l)\mathbf{M}_l^r(\mathbf{z}'))d\mathbf{r} \\ & + ((1-\omega)(z_h'-z_h)\mathbf{M}_h^n(\mathbf{z}') + \omega(z_l'-z_l)\mathbf{M}_l^n(\mathbf{z}'))d\tilde{\mathbf{N}}(\mathbf{z}') \\ & + ((1-\omega)z_h(\mathbf{M}_h^r(\mathbf{z}') - \mathbf{M}_h^r(\mathbf{z})) + \omega z_l(\mathbf{M}_l^r(\mathbf{z}') - \mathbf{M}_l^r(\mathbf{z})))d\mathbf{r} \\ & + ((1-\omega)z_h(\mathbf{M}_h^n(\mathbf{z}') - \mathbf{M}_h^n(\mathbf{z})) + \omega z_l(\mathbf{M}_l^n(\mathbf{z}')) - \mathbf{M}_l^n(\mathbf{z})))d\tilde{\mathbf{N}}(\mathbf{z}) \\ & + \mathbf{M}^{\tilde{n}}(\mathbf{z})(d\tilde{\mathbf{N}}(\mathbf{z}') - d\tilde{\mathbf{N}}(\mathbf{z})) \end{aligned}$$

Defining $\Delta z_i \equiv z_i' - z_i$ and $\Delta\mathbf{M}^x(\mathbf{z}') \equiv \mathbf{M}^x(\mathbf{z}') - \mathbf{M}^x(\mathbf{z})$, we get

$$\begin{aligned} d\mathbf{Y}(\mathbf{z}') - d\mathbf{Y}(\mathbf{z}) =& \left[(1-\omega)\Delta z_h'\mathbf{M}_h^r(\mathbf{z}') + \omega\Delta z_l'\mathbf{M}_l^r(\mathbf{z}')\right]d\mathbf{r} + \left[(1-\omega)\Delta z_h'\mathbf{M}_h^{\tilde{n}}(\mathbf{z}') + \omega\Delta z_l'\mathbf{M}_l^{\tilde{n}}(\mathbf{z}')\right]d\tilde{\mathbf{N}}(\mathbf{z}') \\ & + \left[(1-\omega)z_h\Delta\mathbf{M}_h^r(\mathbf{z}') + \omega z_l\Delta\mathbf{M}_l^r(\mathbf{z}')\right]d\mathbf{r} + \left[(1-\omega)z_h\Delta\mathbf{M}_h^{\tilde{n}}(\mathbf{z}') + \omega z_l\Delta\mathbf{M}_l^{\tilde{n}}(\mathbf{z}')\right]d\tilde{\mathbf{N}}(\mathbf{z}') \\ & + \mathbf{M}^{\tilde{n}}(\mathbf{z})\Big(d\tilde{\mathbf{N}}(\mathbf{z}') - d\tilde{\mathbf{N}}(\mathbf{z})\Big). \end{aligned}$$

B Appendix to HANK zero-liquidity

Solving the zero-liquidity model

This section shows how to derive analytically the main sequence space Jacobian of the zero-liquidity model. This proof follows closely the appendix D.4 of Auclert & Rognlie (2020), but extends it to the case of monetary policy shocks, with different permanent types and a non-homothetic taste for wealth.

B.1 Notation

Let $c_{i,t}(a,e)$ and $a_{i,t+1}(a,e)$ be the policy functions for consumption and savings at time t of a permanent income type i. $c(a,e)$ and $a(a,e)$ are the steady-state policy functions. Similarly, $V_{i,t}(a,e)$ is the value function at time t of a permanent income type i during a transition following an aggregate shock, while $V_i(a,e)$ is the steady state value function.

We denote:

- $c_i'(a,e)$ and $a_i'(a,e)$ the derivative of the steady state policy functions *with respect to* a
- $dc_{i,t}(a,e) = \frac{\partial c_{i,t}(a,e)}{\partial x_s} dx_s$, $da_{i,t}(a,e) = \frac{\partial a_{i,t+1}(a,e)}{\partial x_s} dx_s$ the change in the policy function at time t when there is a shock to a variable at time s
- $V_i'(a,e)$ is the derivative of the steady state value function with respect to a
- $dV_{i,t}(a,e)$ is the change in the value function at t when there is a shock to x at s: $dV_t = \frac{\partial V_{i,t}(a,e)}{\partial x_s} dx_s$

B.2 Solving for the steady-state interest rate

The main idea of the proof is that, in the zero-liquidity limit, all households except the highest productivity type of the high-permanent type will be constrained. Thus, the equilibrium interest rate at the steady state will be such that only this high idiosyncratic type, high permanent productivity type household will be on its Euler equations.

First, recall that the Euler equation in the zero-liquidity model of an agent i with idiosyncratic productivity e_t can be written as

$$(z_i e_t N_t)^{-\sigma} \geq \beta(1+r_t) \sum_{e'} \Pi_{ee'} \left(z_i e_{t+1} N_{t+1}\right)^{-\sigma} + \gamma \zeta^{-\Sigma}$$

Focusing on the steady state, we can write

$$1 \geq \beta(1+r)\rho(e) + \gamma\zeta^{-\Sigma}(z_i e)^{\sigma} \tag{3}$$

where $\rho(e) \equiv \sum_{e'} \Pi_{ee'} \left(\frac{e'}{e}\right)^{-\sigma}$. We assume that $\Pi_{ee'}$ is such that $\rho(e)$ is a strictly increasing function of e, and that $\bar{e} = \arg\max \rho(e)$ is the highest idiosyncratic productivity type[17]. Let us first focus on the case with homothetic preferences, when $\gamma = 0$. In this case, we have

$$1 \geq \beta(1+r)\rho(e).$$

Assume that there exists an $\tilde{e} < \bar{e}$ such that

$$1 = \beta(1+r)\rho(\tilde{e}).$$

Since $\rho(\tilde{e}) < \rho(\bar{e})$, this implies that

$$1 < \beta(1+r)\rho(\bar{e}),$$

[17] This is the case if $\Pi_{e'e}$ approximates an AR(1) process.

which violates 3. Since this holds for all $\tilde{e} < \bar{e}$, if there exists an r such that Equation 3 holds with equality, it must be that

$$1 = (1 + r^*)\beta\rho(\bar{e}).$$

However, since there is no liquidity in the economy and we assume that $a_t \geq 0$ for all households, there exists an infinity of interest rate $r < r^*$ such all households are constrained and the asset market clears. However, as shown by Werning (2015), all those equilibria disappear if we introduce an ε amount of liquidity in the economy. We thus discard them and focus on a steady-state equilibrium where $r = r^*$.

When $\gamma > 0$, note that Equation 3 is an increasing function of both e and z_i. By the same argument, if there exists an r^* such that Equation 3 holds, it must be that it holds for $e = \bar{e}$ and $i = h$. Thus, the equilibrium interest rate is given by

$$1 + r = \frac{1 - \gamma\zeta^{-\Sigma}(z_h\bar{e})^\sigma}{\beta\rho(\bar{e})}.$$

Note that this is a strictly decreasing function of z_h, and the steady-state interest rate will thus decrease when permanent income inequality increases.

B.3 Computing the sequence-space Jacobians $\mathbf{M}^r$ and $\mathbf{A}^r$

We now move on to solving for the main object of interest of this paper, the sequence-space Jacobians for consumption and savings following an interest rate shock $\mathbf{M}^r$, $\mathbf{A}^r$, and an income shock $\mathbf{M}^n$, and $\mathbf{A}^n$.

First, recall that the budget constraint of the household can be written in terms of policy functions as

$$c_{i,t}(a, e) + a_{i,t+1}(a, e) = N_t z_i e + (1 + r_t)a_t. \tag{4}$$

Note that, at the steady state, the market clearing equation implies that we have $a = 0$ for all households. Taking the derivative of Equation 4 with respect to a and evaluating it at the steady state yields

$$\frac{\partial c_i}{\partial a}(0, e) + \lambda_{i,e} = 1 + r.$$

We define as $\lambda_{i,e}$ the slope of the savings policy function of an agent with permanent type i and idiosyncratic productivity e:

$$\lambda_{i,e} \equiv \frac{\partial a_i}{\partial a}(0, e).$$

Note that, for any sequence of shock on r_t, we can take the derivative of 4 with respect to time and evaluate it at zero wealth to obtain

$$dc_{i,t}(0, e) + da_{i,t+1}(0, e) = 0$$

This means that any change in the policy function for consumption at time t following a shock on the interest rate must be perfectly compensated by a change in the policy function for savings. This is intuitive: since there is no wealth in the economy, an interest rate shock has no income effect. If households want to save more, they will have to consume less.

To characterize the consumption response following a sequence of shock, we will study the response of the policy function to each shock separately, and aggregate them at the end.

1. First, let us start with a shock that happens at $t = s$. Since the shock happens only at $t = s$, the value function tomorrow will be equal to the steady state value function. Since an interest rate shock has no income effect, and future wealth is valued at the same rate ($dV_{i,t}(a, e) = 0$), we must have

$$dc_{i,s}(0, e) = 0.$$

This implies that the first column of the Jacobians $\mathbf{M}^r$ and $\mathbf{A}^r$ will be only zeros.

2. When shock happens in the future, that is, when $t < s$, the envelope condition is

$$V'_{i,t}(0, e) = (1 + r_t)u'(c_{i,t}(0, e))$$

Totally differentiating this expression and evaluating it at the steady state yields

$$dV'_{i,t}(0, e) = (1 + r)u''(z_i e)dc_{i,t}(0, e) + u'(z_i e)dr_t. \tag{5}$$

Evaluating this expression at the period of the shock $t = s$, and since $dc_{i,s}(0, e) = 0$, this simplies to

$$dV'_{i,s}(0, e) = u'(z_i e)dr_s.$$

Using the fact that (see derivation in appendix D1 in Auclert et al. (2024))

$$dV'_{i,t}(0, e) = \beta\lambda_{i,e} \sum_{e'} \Pi_{ee'} dV'_{i,t+1}(0, e')$$

We thus have

$$dc_{i,t}(0, e) = \beta\lambda_{i,e} \sum_{e'} \Pi_{ee'} \Big[dc_{i,t+1}(0, e) + \frac{u'(z_i e')}{u''(z_i e)(1 + r)} dr_{t+1}\Big]$$

Note that

$$\frac{u'(z_i e')}{u''(z_i e)} = z_i \frac{(e')^{-\sigma}}{-\sigma(e)^{-\sigma-1}}.$$

But since $\lambda_{i,e} = 0$ (constrained households consume all marginal additional unit of wealth) for all $e \neq \bar{e}$, we have

$$dc_{i,t}(0,\bar{e}) = \beta\lambda_{i,\bar{e}} dc_{i,t+1}(0,\bar{e}) + z_i \sum_{e'} \Pi_{\bar{e}e'} \frac{u'(e')}{u''(\bar{e})(1+r)} dr_{t+1}.$$

Let us define

$$\rho(\bar{e}) \equiv \sum_{e'} \Pi_{\bar{e}e'} \left(\frac{e'}{\bar{e}}\right)^{-\sigma}$$

Note that, when the variance of shocks tends to 0 and the auto-correlation of shock tends to 1, $K^r \to 1$.

We can then write

$$dc_{i,t}(0,\bar{e}) = \beta\lambda_{i,\bar{e}} dc_{i,t+1}(0,\bar{e}) - z_i \frac{\bar{e}}{\Pi_{\bar{e}\bar{e}}} \rho(\bar{e}) \frac{1}{\sigma(1+r)} dr_{t+1}.$$

Solving this equation forward, we obtain that for any shock at $s > t$ (using the fact that $dr_{t+1} = 0$ except when $t+1 = s$):

$$dc_{i,t}(0,\bar{e}) = -(\beta\lambda_{i,\bar{e}}\Pi_{\bar{e}\bar{e}})^{s-t} \left(z_i \frac{\bar{e}}{\Pi_{\bar{e}\bar{e}}} \rho(\bar{e}) \frac{1}{\sigma(1+r)} \right) dr_s \quad \text{for } s > t.$$

Summing for all potential shocks, we get

$$dc_{i,t}(0,\bar{e}) = -\sum_{s>t} (\beta\lambda_{i,\bar{e}}\Pi_{\bar{e}\bar{e}})^{s-t} \left(z_i \frac{\bar{e}}{\Pi_{\bar{e}\bar{e}}} \rho(\bar{e}) \frac{1}{\sigma(1+r)} \right) dr_s \quad \text{for } s > t$$

And using the fact that $da_{t+1}(0,\bar{e}) = -dc_t(0,\bar{e})$, we obtain

$$da_{i,t+1}(0,\bar{e}) = \sum_{s>t} (\beta\lambda_{i,\bar{e}}\Pi_{\bar{e}\bar{e}})^{s-t} \left(z_i \frac{\bar{e}}{\Pi_{\bar{e}\bar{e}}} \rho(\bar{e}) \frac{1}{\sigma(1+r)} \right) dr_s \quad \text{for } s > t$$

which we can rewrite as

$$da_{i,t+1}(0,\bar{e}) = \sum_{s>t} (\beta\lambda_i)^{s-t} \left(z_i \frac{\bar{e}}{\Pi_{\bar{e}\bar{e}}} \rho(\bar{e}) \frac{1}{\sigma(1+r)} \right) dr_s \quad \text{for } s > t$$

where $\lambda_i \equiv \lambda_{i,\bar{e}}\Pi_{\bar{e}\bar{e}}$.

Aggregation.

We compute the aggregate supply of savings of permanent type i at time t. The savings at time t for an unconstrained household of permanent type i that has been unconstrained for the last $\tau - 1$ periods is:

$$a_{i,\tau,t+1} = a_{i,t+1}(a_{i,\tau-1,t}, \bar{e})$$

Where we define $a_{i,-1,0} = 0$ the wealth holdings of all constrained households whatever their permanent types.

Totally differentiating the expression yields

$$da_{i,\tau,t+1} = \underbrace{da_{i,t+1}(0,\bar{e})}_{\text{change in policy function}} + \lambda_{i,\bar{e}} \underbrace{da_{i,\tau-1,t}}_{\text{change in wealth holdings}}$$

Aggregating with weights $\pi_{\bar{e}}(1 - \Pi_{\bar{e}\bar{e}})\Pi^{\tau}_{\bar{e}\bar{e}}$ and

$$\sum_{\tau=0}^{\infty} \pi_{\bar{e}} \left(1 - \Pi_{\bar{e}\bar{e}}\right) \Pi^{\tau}_{\bar{e}\bar{e}} da_{i,\tau,t+1} = \sum_{\tau=0}^{\infty} \pi_{\bar{e}} \left(1 - \Pi_{\bar{e}\bar{e}}\right) \Pi^{\tau}_{\bar{e}\bar{e}} (da_{i,t+1}(\bar{e}, 0) + \lambda_{i,\bar{e}} da_{i,\tau-1,t})$$

using that $A_{i,t+1} = \sum_{\tau=0}^{\infty} \pi_{\bar{e}} \left(1 - \Pi_{\bar{e}\bar{e}}\right) \Pi^{\tau}_{\bar{e}\bar{e}} a_{i,\tau,t+1}$, we get

$$dA_{i,t+1} = \sum_{\tau=0}^{\infty} \pi_{\bar{e}} \left(1 - \Pi_{\bar{e}\bar{e}}\right) \Pi^{\tau}_{\bar{e}\bar{e}} (da_{i,t+1}(0, \bar{e}) + \lambda_{i,\bar{e}} da_{i,\tau-1,t})$$

$$dA_{i,t+1} = da_{i,t+1}(0, \bar{e}) \sum_{\tau=0}^{\infty} \pi_{\bar{e}} \left(1 - \Pi_{\bar{e}\bar{e}}\right) \Pi^{\tau}_{\bar{e}e} + \sum_{\tau=0}^{\infty} \pi_{\bar{e}} \left(1 - \Pi_{\bar{e}\bar{e}}\right) \Pi^{\tau}_{\bar{e}\bar{e}} \lambda_{i,\bar{e}} da_{i,\tau-1,t}$$

$$dA_{i,t+1} = \pi_{\bar{e}} da_{i,t+1}(0, \bar{e}) + \sum_{\tau=0}^{\infty} \pi_{\bar{e}} \left(1 - \Pi_{\bar{e}\bar{e}}\right) \Pi^{\tau}_{\bar{e}e} \lambda_{i,\bar{e}} da_{i,\tau-1,t}$$

Noting that $da_{i,-1,t} = 0$, we get

$$dA_{i,t+1} = \pi_{\bar{e}} da_{i,t+1}(0, \bar{e}) + \sum_{\tau=0}^{\infty} \pi_{\bar{e}} \left(1 - \Pi_{\bar{e}\bar{e}}\right) \Pi^{\tau}_{\bar{e}e} \lambda_{i,\bar{e}} da_{i,\tau,t}$$

Note that that second term can be written as

$$\sum_{\tau=0}^{\infty} \pi_{\bar{e}} \left(1 - \Pi_{\bar{e}e}\right) \Pi^{\tau}_{\bar{e}e} \lambda_{i,\bar{e}} da_{i,\tau} = \Pi_{\bar{e}e} \lambda_{i,\bar{e}} dA_{i,t}$$

so that we have

$$\begin{aligned} dA_{i,t+1} &= da_{i,t+1}(0, \bar{e}) \pi_{\bar{e}} + \lambda_{i,\bar{e}} \Pi_{\bar{e}\bar{e}} dA_{i,t} \\ &= da_{i,t+1}(0, \bar{e}) \pi_{\bar{e}} + \lambda_i dA_{i,t} \end{aligned}$$

From the previous subsection, we have

$$da_{i,t+1}(0,\bar{e}) = \sum_{s>t}(\beta\lambda_i)^{s-t}\left(z_i\frac{\bar{e}}{\Pi_{\bar{e}\bar{e}}}\rho(\bar{e})\frac{1}{\sigma(1+r)}\right)dr_s \quad \text{for } s>t$$

Combining with the law of motion, we get

$$dA_{i,t+1} = \pi_{\bar{e}}\sum_{s>t}(\beta\lambda_i)^{s-t}\left(z_i\frac{\bar{e}}{\Pi_{\bar{e}\bar{e}}}\rho(\bar{e})\frac{1}{\sigma(1+r)}\right)dr_s + \lambda_i dA_{i,t}$$

Defining the effective share of unconstrained household $1-\mu \equiv \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}$,

$$dA_{i,t+1} = (1-\mu)\sum_{s>t}(\beta\lambda_i)^{s-t}\left(z_i\rho(\bar{e})\frac{1}{\sigma(1+r)}\right)dr_s + \lambda_i dA_{i,t}$$

For an unexpected shock at $s=0$, we have

$$dA_{i,t+1} = 0$$

the first column is only zero.

For a shock at $s=1$, we have in $t=0$,

$$dA_{i,1} = (1-\mu)\beta\lambda_i\left(\rho(\bar{e})\frac{1}{\sigma(1+r)}z_i\right)dr_1$$

And,

$$dA_1 = \sum_i \omega_i dA_{i,1} = (1-\mu)\beta\sum_i \omega_i\lambda_i\left(\rho(\bar{e})\frac{1}{\sigma(1+r)}z_i\right)dr_1$$

$$\mathbf{A}^r = (1-\mu)\sum_i \omega_i z_i \mathbf{T}_i^r(a_+)\mathbf{T}_i^r(a_-)$$

$$\mathbf{T}_i^r(a_+) = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda_i & 1 & 0 & \cdots \\ \lambda_i^2 & \lambda_i & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \text{ and } \mathbf{T}_i^r(a_-) = \begin{pmatrix} 0 & \frac{\beta\lambda_i}{1+r}\frac{\rho(\bar{e})}{\sigma} & \frac{(\beta\lambda_i)^2}{1+r}\frac{\rho(\bar{e})}{\sigma} & \cdots \\ 0 & 0 & \frac{\beta\lambda_i}{1+r}\frac{\rho(\bar{e})}{\sigma} & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

B.4 Solving for λ_i

The key parameter that determines the behavior of households is λ_i. To solve for it, we use the Euler equation of the highest permanent type h with the highest idiosyncratic type $\bar{e}$ at the steady state

$$c_h\left(a_-,\bar{e}\right)^{-\sigma} = \beta(1+r)\sum_{e'}\Pi_{\bar{e}e'}\left(c_h\left(a_h\left(a_-,\bar{e}\right),e'\right)\right)^{-\sigma} + \gamma(a_h\left(a_-,\bar{e}\right)+\zeta)^{-\Sigma}$$

Differentiating with respect to a_- and evaluating at $a_- = 0$:

$$u''(z_h\bar{e})\frac{\partial c_h}{\partial a_-}(0,\bar{e}) = \beta(1+r)\sum_{e'}\Pi_{\bar{e}e'}u''\left(z_h e'\right)\frac{\partial c_h}{\partial a_h}\left(0,e'\right)\frac{\partial a_h}{\partial a_-}(0,\bar{e}) - \gamma\Sigma(a_h\left(0,\bar{e}\right)+\zeta)^{-\Sigma-1}\frac{\partial a_h}{\partial a_-}(0,\bar{e})$$

Using that $\frac{\partial c_h}{\partial a_-}(0,\bar{e}) = (1+r)m_{h,\bar{e}}$ and $\frac{\partial a_h}{\partial a_-}(0,\bar{e}) = (1+r)(1-m_{h,\bar{e}})$, at the steady state $a_h\left(0,\bar{e}\right) = 0$ and,

$$u''(z_h\bar{e})m_{h,\bar{e}} = \beta(1+r)^2\sum_{e'}\Pi_{\bar{e}e'}u''\left(z_h e'\right)m_{h,e'}\cdot(1-m_{h,\bar{e}}) - \gamma\Sigma(1-m_{h,\bar{e}})\zeta^{-\Sigma-1}$$

$$-\sigma(z_h\bar{e})^{-\sigma-1}m_{h,\bar{e}} = -\sigma\beta(1+r)^2\sum_{e'}\Pi_{\bar{e}e'}(z_h e')^{-\sigma-1}m_{h,e'}\cdot(1-m_{h,\bar{e}}) - \gamma\Sigma(1-m_{h,\bar{e}})\zeta^{-\Sigma-1}$$

$$m_{h,\bar{e}} = \beta(1+r)^2\sum_{e'}\Pi_{\bar{e}e'}\left(\frac{e'}{\bar{e}}\right)^{-\sigma-1}m_{h,e'}\cdot(1-m_{h,\bar{e}}) + \gamma\frac{\Sigma}{\sigma}(1-m_{h,\bar{e}})\zeta^{-\Sigma-1}(z_h\bar{e})^{\sigma+1}$$

Using that $\forall e' \neq \bar{e}, \quad m_{h,e'} = 1$,

$$m_{h,\bar{e}} = \beta(1+r)^2\left(\sum_{e'\neq\bar{e}}\Pi_{\bar{e}e'}\left(\frac{e'}{\bar{e}}\right)^{-\sigma-1} + \Pi_{\bar{e}\bar{e}}m_{h,\bar{e}}\right)\cdot(1-m_{h,\bar{e}}) + \gamma\frac{\Sigma}{\sigma}(1-m_{h,\bar{e}})\zeta^{-\Sigma-1}(z_h\bar{e})^{\sigma+1}$$

$$\frac{m_{h,\bar{e}}}{1-m_{h,\bar{e}}} = \beta(1+r)^2\left(\sum_{e'\neq\bar{e}}\Pi_{\bar{e}e'}\left(\frac{e'}{\bar{e}}\right)^{-\sigma-1} + \Pi_{\bar{e}\bar{e}}m_{h,\bar{e}}\right) + \gamma\frac{\Sigma}{\sigma}\zeta^{-\Sigma-1}(z_h\bar{e})^{\sigma+1}$$

This is a quadratic equation that pins down the MPC $m_{h,\bar{e}}$ of the highest permanent type with idiosyncratic type $\bar{e}$. Notice that, when there is no taste for wealth, $\gamma = 0$, the equation no longer depends on z_h and $\forall i, \quad m_{i,\bar{e}} = m_{\bar{e}}$. From $m_{h,\bar{e}}$, we recover λ_h since:

$$\lambda_h = \Pi_{\bar{e}\bar{e}}(1+r)(1-m_{h,\bar{e}})$$

Since there is always one unique λ whatever the preference, we can drop the type subscript and have $\lambda \equiv \lambda_h$.

B.5 Rewritting the sequence-space Jacobians

Writing the sequence-space Jacobian of the savings reponse following an interest rate shock:

$$\mathbf{A}^r = (1-\mu)\rho(\bar{e})\frac{\beta\lambda_i}{1+r}\frac{1}{\sigma}\sum_i \omega_i z_i \mathbf{T}_i^r(a_+)\mathbf{T}_i^r(a_-)$$

$$\mathbf{T}_i^r(a_+) = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda_i & 1 & 0 & \cdots \\ \lambda_i^2 & \lambda_i & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \text{ and } \mathbf{T}_i^r(a_-) = \begin{pmatrix} 0 & 1 & \beta\lambda_i & \cdots \\ 0 & 0 & 1 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

When there is no taste for wealth, there is always a unique λ common across permanent types, we can get rid of the i subscript in the $\mathbf{T}$ matrices and: A

$$\mathbf{A}^r = (1-\mu)\rho(\bar{e})\frac{\beta\lambda}{1+r}\frac{1}{\sigma}\sum_i \omega_i z_i \mathbf{T}^r(a_+)\mathbf{T}^r(a_-) = (1-\mu)\rho(\bar{e})\frac{\beta\lambda}{1+r}\frac{1}{\sigma}\mathbf{T}^r(a_+)\mathbf{T}^r(a_-)$$

When there is a taste for wealth, there is still one λ for the highest permanent type, all other λs are 0, and:

$$\mathbf{A}^r = (1-\mu)\rho(\bar{e})\frac{\beta\lambda}{1+r}\frac{1}{\sigma}z_h\omega_h\mathbf{T}^r(a_+)\mathbf{T}^r(a_-)$$

$$\mathbf{A}^r = (1-\mu)\rho(\bar{e})\frac{\beta\lambda}{1+r}\frac{1}{\sigma}z_h\omega_h \begin{pmatrix} 0 & 1 & \beta\lambda & \cdots \\ 0 & \lambda & 1+\beta\lambda^2 & \cdots \\ 0 & \lambda^2 & \lambda^2\beta\lambda+\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$da_1 = \beta\lambda dr_2, dc_0 = -\beta\lambda dr_2$$

$$da_2 = (1+\beta\lambda)dr_2$$

$$c_2 + a_2 = (1+r)a_1 + N \iff dc_2 = (1+r)da_1 - da_2 = (1+r)\beta\lambda dr_2 - (1+\beta\lambda)dr_2$$

$$\iff dc_2 = [(1+r)\beta\lambda - (1+\beta\lambda)]dr_2$$

From $\mathbf{A}^r$, we recover $\mathbf{M}^r$ using the vectorized budget constraint:

$$\mathbf{M}^r + \mathbf{A}^r = (1+r)\mathbf{L}\mathbf{A}^r \iff \mathbf{M}^r = ((1+r)\mathbf{L} - \mathbf{I})\mathbf{A}^r$$

Starting from the non-homothetic case:

$$\mathbf{M}^r = (1-\mu)\rho(\bar{e})z_h\omega_h\frac{\beta\lambda}{1+r}\frac{1}{\sigma}\begin{pmatrix} -1 & 0 & 0 & \cdots \\ 1+r & -1 & 0 & \cdots \\ 0 & 1+r & -1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}\begin{pmatrix} 0 & 1 & \beta\lambda & \cdots \\ 0 & \lambda & 1+\beta\lambda^2 & \cdots \\ 0 & \lambda^2 & \lambda^2\beta\lambda+\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathbf{M}^r = (1-\mu)\rho(\bar{e})z_h\omega_h \frac{\beta\lambda}{1+r}\frac{1}{\sigma}\begin{pmatrix} 0 & -1 & -\beta\lambda & \cdots \\ 0 & 1+r-\lambda & (1+r)\beta\lambda - 1 - \beta\lambda & \cdots \\ 0 & (1+r)\lambda - \lambda^2 & (1+r)(1+\beta\lambda^2) - \lambda^2\beta\lambda - \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Finishing with the homothetic case:

$$\mathbf{M}^r = (1-\mu)\rho(\bar{e})\frac{\beta\lambda}{1+r}\frac{1}{\sigma}\begin{pmatrix} 0 & -1 & -\beta\lambda & \cdots \\ 0 & 1+r-\lambda & (1+r)\beta\lambda - 1 - \beta\lambda & \cdots \\ 0 & (1+r)\lambda - \lambda^2 & (1+r)(1+\beta\lambda) - \lambda^2\beta\lambda - \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

B.6 Computing the sequence-space Jacobians $\mathbf{M}^n$ and $\mathbf{A}^n$

We follow closely Auclert et al. (2024), appendix D4, with the introduction of permanent types. Differentiating the budget constraint, we have that:

$$dc_{i,t}(0,e) = m_e z_i dN_t$$

We have that

$$dV'_{i,t}(0,e) = \beta\lambda_e \sum_{e'} \Pi_{ee'} dV'_{i,t+1}(0,e')$$

Computing the Envelope condition

$$V'_{i,t}(a_-,e) = (1+r)u'(c_{i,t}(a_-,e))$$

Differentiating and evaluating it at the steady state,

$$dV'_{i,t}(0,e) = (1+r)u''(ez_iN)dc_{i,t}(0,e)$$

Plugging it in the equation above:

$$(1+r)u''(ez_iN)dc_{i,t}(0,e) = \beta\lambda_e \sum_{e'} \Pi_{ee'}(1+r)u''(e'z_iN)dc_{i,t+1}(0,e')$$

$$dc_{i,t}(0,e) = \beta\lambda_e \sum_{e'} \Pi_{ee'} \left(\frac{e'z_iN}{ez_iN}\right)^{-\sigma-1} dc_{i,t+1}(0,e')$$

$$dc_{i,t}(0,e) = \beta\lambda_e \sum_{e'} \Pi_{ee'} \left(\frac{e'}{e}\right)^{-\sigma-1} dc_{i,t+1}(0,e')$$

The change in the policy function for consumption of permanent type i at time t due to shock on N_s

$$dc_{i,t}(0,e) = \begin{cases} (\beta\lambda_{\bar{e}}\Pi_{\bar{e}\bar{e}})^{s-t} \left(\sum_{e'\neq\bar{e}} \frac{\Pi_{\bar{e}e'}}{\Pi_{\bar{e}\bar{e}}} \left(\frac{e'}{\bar{e}}\right)^{-(\sigma+1)} \cdot e' + m_{i,\bar{e}}\bar{e}\right) z_i dN_s & s > t, e = \bar{e} \\ m_{i,\bar{e}}\bar{e} z_i dN_t & s = t, e = \bar{e} \\ 0 & s < t \end{cases}$$

Adding up across all shocks $\{N_s\}$,

$$dc_{i,t}(0,\bar{e}) = m_{i,\bar{e}}\bar{e} z_i dN_t + \left(\sum_{e'\neq\bar{e}} \frac{\Pi_{\bar{e}e'}}{\Pi_{\bar{e}\bar{e}}} \left(\frac{e'}{\bar{e}}\right)^{-(\sigma+1)} \cdot e' + m_{i,\bar{e}}\bar{e}\right) \sum_{s>t}^{\infty} (\beta\lambda_{\bar{e}}\Pi_{\bar{e}\bar{e}})^{s-t} z_i dN_s$$

$$da_{i,t+1}(0,\bar{e}) = (1-m_{i,\bar{e}})\bar{e} z_i dN_t - \bar{e}\left(\sum_{e'\neq\bar{e}} \frac{\Pi_{\bar{e}e'}}{\Pi_{\bar{e}\bar{e}}} \left(\frac{e'}{\bar{e}}\right)^{-\sigma} + m_{i,\bar{e}}\right) \sum_{s>t}^{\infty} (\beta\lambda_{\bar{e}}\Pi_{\bar{e}\bar{e}})^{s-t} z_i dN_s$$

B.7 Aggregation

As before, we have that,

$$dA_{i,t+1} = da_{i,t+1}(0,\bar{e})\pi_{\bar{e}} + \lambda_i dA_{i,t}$$

Combining the last two equations and defining $K \equiv \sum_{e'\neq\bar{e}} \frac{\Pi_{\bar{e}e'}}{\Pi_{\bar{e}\bar{e}}} \left(\frac{e'}{\bar{e}}\right)^{-\sigma}$, we get:

$$dA_{i,t+1} = \pi_{\bar{e}}(1-m_{i,\bar{e}})\bar{e} z_i dN_t - \pi_{\bar{e}}\bar{e}\,(K + m_{i,\bar{e}}) \sum_{s>t}^{\infty} (\beta\lambda_{\bar{e}}\Pi_{\bar{e}\bar{e}})^{s-t} z_i dN_s + \lambda_i dA_{i,t}$$

Summing across permanent types given that $dA_{t+1} = \sum_i \omega_i dA_{i,t+1}$, we get the following aggregate sequence-space Jacobian:

$$\mathbf{A}^n = \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}} \sum_i \omega_i z_i \mathbf{T}_i^n(a_+)\mathbf{T}_i^n(a_-)$$

$$\text{With } \mathbf{T}_i^n(a_+) = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda_i & 1 & 0 & \cdots \\ \lambda_i^2 & \lambda_i & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\text{and } \mathbf{T}_i^n(a_-) = \begin{pmatrix} \frac{\lambda_i}{1+r} & -\left(\frac{1}{\beta(1+r)} - \frac{\lambda_i}{1+r}\right)(\beta\lambda) & -\left(\frac{1}{\beta(1+r)} - \frac{\lambda_i}{1+r}\right)(\beta\lambda_i)^2 & \cdots \\ 0 & \frac{\lambda_i}{1+r} & -\left(\frac{1}{\beta(1+r)} - \frac{\lambda_i}{1+r}\right)(\beta\lambda_i) & \cdots \\ 0 & 0 & \frac{\lambda_i}{1+r} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

B.8 Rewritting the sequence-space Jacobian for income shocks

As before, when there is no preference for wealth, the sequence-space Jacobian is given by:

$$\mathbf{A}^n = \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}\frac{1}{1+r}\mathbf{T}^n(a_+)\mathbf{T}^n(a_-).$$

When we have preference for wealth,

$$\mathbf{A}^n = \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}\frac{\omega_h z_h}{1+r}\mathbf{T}^n(a_+)\mathbf{T}^n(a_-).$$

$$\text{With } \mathbf{T}^n(a_+) = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda & 1 & 0 & \cdots \\ \lambda^2 & \lambda & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\text{and } \mathbf{T}^n(a_-) = \begin{pmatrix} \lambda & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2 & \cdots \\ 0 & \lambda & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \cdots \\ 0 & 0 & \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathbf{A}^n = \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}\frac{\omega_h z_h}{1+r}\begin{pmatrix} \lambda & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2 & \cdots \\ \lambda^2 & -\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda & -\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2-\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \cdots \\ \lambda^3 & -\lambda^2\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda^2 & -\lambda^2\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2-\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

From $\mathbf{A}_h^n = \mathbf{A}^n/(\omega_h z_h)$, we recover $\mathbf{M}_h^n$ using the vectorized budget constraint:

$$\mathbf{M}_h^n + \mathbf{A}_h^n = (1+r)\mathbf{L}\mathbf{A}_h^n + \mathbf{I} \iff \mathbf{M}_h^n = ((1+r)\mathbf{L}-\mathbf{I})\mathbf{A}_h^n + \mathbf{I}$$

Defining

$$\tilde{\mathbf{M}}_h^n = \begin{pmatrix} -\lambda & \left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2 & \cdots \\ (1+r)\lambda-\lambda^2 & (\lambda-(1+r))\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)-\lambda & (\beta\lambda^2-(1+r)\beta\lambda+1)\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \cdots \\ (1+r)\lambda^2-\lambda^3 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathbf{M}_h^n = \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}\frac{1}{1+r}\tilde{\mathbf{M}}_h^n + \mathbf{I}$$

And:

$$\mathbf{M}^n = \omega_h z_h \mathbf{M}_h^n + \omega_l z_l \mathbf{I} = \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}} \frac{\omega_h z_h}{1+r} \tilde{\mathbf{M}}^n + \mathbf{I}$$

The first line, first column of $\mathbf{M}_h^n$ is the static MPC :

$$1 - \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}} \frac{\lambda}{1+r} = \underbrace{(1 - \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}})}_{\text{share of constrained within high type}} \underbrace{1}_{\text{MPC HtM}} + \underbrace{\frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}}_{\text{share of Ricardian within high type}} \underbrace{(1 - \frac{\lambda}{1+r})}_{\text{MPC Ricardian}}$$

While the following elements on the first column give us the iMPC:

$$\underbrace{\frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}}_{\text{share of Ricardian}} \underbrace{\lambda^t}_{\text{rate of decay}} \underbrace{(1 - \frac{\lambda}{1+r})}_{\text{static MPC}}$$

B.9 Aggregate Intertemporal Keynesian Cross

Differentiating the goods market clearing condition, we have that:

$$d\mathbf{Y} = (1-\omega) z_h \mathbf{M}_h^n d\mathbf{N} + (1-\omega) z_h \mathbf{M}_h^r d\mathbf{r} + \omega z_l \mathbf{M}_l^r d\mathbf{r} + z_l \omega \mathbf{M}_l^n d\mathbf{N}$$

In the non-homothetic case, we have that:

$$d\mathbf{Y} = (1-\omega) z_h \mathbf{M}_h^n d\mathbf{N} + (1-\omega) z_h \mathbf{M}_h^r d\mathbf{r} + z_l \omega \mathbf{I} d\mathbf{N}$$

And the IKC is:

$$(\mathbf{I} - (1-\omega) z_h \mathbf{M}_h^n - z_l \omega \mathbf{I}) d\mathbf{Y} = (1-\omega) z_h \mathbf{M}_h^r d\mathbf{r}$$

$$\iff d\mathbf{Y} = [\mathbf{K}(\mathbf{I} - (1-\omega) z_h \mathbf{M}_h^n - z_l \omega \mathbf{I})]^{-1} \mathbf{K} (1-\omega) z_h \mathbf{M}_h^r d\mathbf{r}$$

$$\iff d\mathbf{Y} = \underbrace{[(1-\omega) z_h \mathbf{K}(\mathbf{I} - \mathbf{M}_h^n)]^{-1} \mathbf{K}}_{\text{GE amplification}} \underbrace{(1-\omega) z_h \mathbf{M}_h^r d\mathbf{r}}_{\text{Direct effect}}$$

Notice that, with non-homothetic preferences, an increase in permanent income inequality increases the direct effect and decreases the indirect effect. However, at the aggregate level, those two effects cancel out.

$$\iff d\mathbf{Y} = [\mathbf{K}(\mathbf{I} - \mathbf{M}_h^n)]^{-1} \mathbf{K} \mathbf{M}_h^r d\mathbf{r}$$

In the homothetic case, sequence-space Jacobians do not depend on permanent types and we can drop the subscript for types:

$$d\mathbf{Y} = \mathbf{M}^n d\mathbf{N} + \mathbf{M}^r d\mathbf{r}$$

$$d\mathbf{Y} = [\mathbf{K}(\mathbf{I} - \mathbf{M}^n)]^{-1} \mathbf{K} \mathbf{M}^r d\mathbf{r}$$

With homothetic preferences, an increase in permanent income inequality has no effect on the weight of the direct and indirect effect and so no effect at the aggregate level.

B.10 Aggregate effect of monetary policy shock (proof 4)

To compute the aggregate effect of a monetary policy shock on output, we need to start from the vectorized asset market clearing condition:

$$\mathbf{A}(\{r_t, N_t\}) = \mathbf{0}.$$

Totally differentiating the equation, we get that:

$$\mathbf{A}^r d\mathbf{r} + \mathbf{A}^n d\mathbf{N} = 0$$

In equilibrium, $d\mathbf{N} = d\mathbf{Y}$ and the aggregate effect of a monetary policy shock is given by

$$d\mathbf{Y} = -(\mathbf{A}^n)^{-1}\mathbf{A}^r d\mathbf{r}. \tag{6}$$

Remember that those two aggregate sequence-space Jacobians can be written as the product of two Toeplitz matrices (all diagonal elements are equal):

$$\mathbf{A}^n = \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}\frac{\omega_h z_h}{1+r}\mathbf{T}^n(a_+)\mathbf{T}^n(a_-),$$

With $\mathbf{T}^n(a_+) = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda & 1 & 0 & \cdots \\ \lambda^2 & \lambda & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$ and $\mathbf{T}^n(a_-) = \begin{pmatrix} \lambda & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2 & \cdots \\ 0 & \lambda & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \cdots \\ 0 & 0 & \lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$.

And:

$$\mathbf{A}^r = \frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}\rho(\bar{e})\frac{\beta\lambda}{\sigma(1+r)}\omega_h z_h \mathbf{T}^r(a_+)\mathbf{T}^r(a_-),$$

With:

$$\mathbf{T}^r(a_+) = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \lambda & 1 & 0 & \cdots \\ \lambda^2 & \lambda & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \text{ and } \mathbf{T}^r(a_-) = \begin{pmatrix} 0 & 1 & \beta\lambda & \cdots \\ 0 & 0 & 1 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Using that Toeplitz structure to solve the aggregate effect of a monetary policy shock in equation 6,

$$d\mathbf{Y} = -\beta\lambda\frac{1}{\sigma}\rho(\bar{e})\mathbf{T}^n(a_-)^{-1}\mathbf{T}^n(a_+)^{-1}\mathbf{T}^r(a_+)\mathbf{T}^r(a_-)d\mathbf{r}.$$

Noticing that $\mathbf{T}^n(a_+) = \mathbf{T}^r(a_+)$, we get that,

$$d\mathbf{Y} = -\beta\lambda\frac{1}{\sigma}\rho(\bar{e})\mathbf{T}^n(a_-)^{-1}\mathbf{T}^r(a_-)d\mathbf{r}.$$

Notice that any Toeplitz matrix can be summarised by its symbol. For $\mathbf{T}^n(a_-)$, its symbol is given by:

$$g_n(z) = \lambda - \sum_{k=1}^{\infty}(\frac{1}{\beta} - \lambda)(\beta\lambda z^{-1})^k \iff g_n(z) = \lambda - (\frac{1}{\beta} - \lambda)\frac{\beta\lambda z^{-1}}{1-\beta\lambda z^{-1}}.$$

The symbol associated to $\mathbf{T}^r(a_-)$ is given by:

$$g_r(z) = \sum_{k=1}^{\infty}(\beta\lambda z^{-1})^k = \frac{\beta\lambda z^{-1}}{1-\beta\lambda z^{-1}}.$$

Computing the product of the symbols of the two Toeplitz matrices gives us the symbol associated to the product of those two matrices:

$$g_n(z)^{-1}g_r(z) = \left(\lambda - (\frac{1}{\beta} - \lambda)\frac{\beta\lambda z^{-1}}{1-\beta\lambda z^{-1}}\right)^{-1}\frac{\beta\lambda z^{-1}}{1-\beta\lambda z^{-1}} = \frac{\beta z^{-1}}{1-z^{-1}} = \beta\sum_{k=1}(z^{-1})^k.$$

Using that symbol, we compute the first element of the $d\mathbf{Y}$ vector

$$dY_0 = -\beta\frac{1}{\sigma}\rho(\bar{e})\frac{\rho}{1-\rho}dr_0 \text{ with } \rho(\bar{e}) = \sum_{e'}\Pi_{\bar{e}e'}\left(\frac{e'}{\bar{e}}\right)^{-\sigma}.$$

C Direct and Indirect Effects

C.1 Direct effect

The direct effect of a monetary policy shock is given by:

$$\text{Direct effect} = -\mathbf{M}^r d\mathbf{r} = -\frac{\pi_{\bar{e}}\bar{e}}{\Pi_{\bar{e}\bar{e}}}\rho(\bar{e})\omega_h z_h\frac{1}{\sigma}\sum_{t=1}^{\infty}\frac{(\beta\lambda)^t}{1+r}\rho^t dr_0 = -(1-\mu)\rho(\bar{e})\omega_h z_h\frac{1}{1+r}\frac{1}{\sigma}\frac{\rho\lambda\beta}{1-\rho\lambda\beta}dr_0.$$

Computing the derivative of the direct effect with respect to the level of inequality,

$$\frac{d|\text{Direct effect}|}{dz_h} = \underbrace{\frac{\text{Direct effect}}{z_h}}_{\text{composition effect}>0} \underbrace{+(1-\mu)\rho(\bar{e})\omega_h z_h\frac{1}{\sigma}\rho\beta dr_0\left[\frac{d\lambda}{dz_h}\frac{1+\rho\beta(1-\lambda)}{(1-\rho\lambda\beta)^2} - \frac{\frac{dr}{dz_h}}{(1+r)^2}\right]}_{\text{behavior effect}<0}.$$

Controlling for the composition effect, the direct effect is negative.

C.2 EIS high permanent/idiosyncratic type

To compute the EIS, we need to compute the expected ratio of the consumption of the high p/i (permanent/idiosyncratic) type in $t = 1$ over the consumption of the high p/i type in $t = 0$ conditional on the household being a high p/i type in $t = 0$ when there is an expected shock on the real interest rate in $t = 1$.

Starting with the change in the policy function of the high p/i type in $t = 0$ given that there is a real interest rate shock in $t = 1$:

$$dc_{h,0}(0, \bar{e}) = -(\beta\lambda_{h,\bar{e}})\left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma(1 + r)}\right) dr_1$$

And, given that the shock has just been announced, the consumption at $t = 0$ is just given by the change in the policy function in $t = 0$:

$$c_{h,0} = \bar{e}z_h - (\beta\lambda_{h,\bar{e}})\left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma(1 + r)}\right) dr_1$$

Computing the savings of the high p/i (permanent/idiosyncratic) type:

$$a_{h,1} + c_{h,0} = (1 + r)a_h^{ss} + z_h\bar{e}N_0$$

Which simplifies to:

$$a_{h,1} = -c_{h,0} + z_h\bar{e} = -dc_{h,0}(0, \bar{e})$$

There is no change in policy function at $t = 1$ since the shock is contemporaneous, and we thus have

$$dc_{h,1}(0, \bar{e}) = 0$$

However, conditional on staying unconstrained, the actual consumption level of the household will change since it has accumulated some wealth at the previous period. The increase in consumption $dc_{h,1}$ will be

$$dc_{h,1} = m_{\bar{e}}(1 + r + dr_1)da_{h,1}(0, \bar{e}) = -m_{\bar{e}}(1 + r + dr_1)dc_{h,0}(0, \bar{e}).$$

Taking into account the expectation with respect to idiosyncratic shock, we obtain

$$\mathbb{E}[c_{h,1}|e = \bar{e}] = \mathbb{E}[e'z_h - m_{e'}(1 + r + dr_1)dc_{h,0}(0, \bar{e})]$$

Noting that $m_{e'} = 1$ if $e' \neq \bar{e}$, we can rewrite this as

$$\mathbb{E}[c_{h,1}|e = \bar{e}] = z_h \mathbb{E}[e'|\bar{e}] - (1 + r + dr_1)\Big(\Pi_{\bar{e}\bar{e}} m_{\bar{e}} + (1 - \Pi_{\bar{e}\bar{e}})\Big) dc_{h,0}(0, \bar{e})$$

We can then compute the ratio of the two consumption levels as

$$\frac{\mathbb{E}[c_{h,1}]}{c_{h,0}} = \frac{\mathbb{E}[e'|\bar{e}] z_h + (1 + r + dr_1)\Big(\Pi_{\bar{e}\bar{e}} m_{\bar{e}} + (1 - \Pi_{\bar{e}\bar{e}})\Big) (\beta\lambda_{h,\bar{e}}) \left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma(1+r)}\right) dr_1}{\bar{e} z_h - (\beta\lambda_{h,\bar{e}}) \left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma(1+r)}\right) dr_1}$$

Getting rid of the second-order term, we obtain:

$$\frac{\mathbb{E}[c_{h,1}]}{c_{h,0}} = \frac{\mathbb{E}[e'|\bar{e}] z_h + \Big(\Pi_{\bar{e}\bar{e}} m_{\bar{e}} + (1 - \Pi_{\bar{e}\bar{e}})\Big) (\beta\lambda_{h,\bar{e}}) \left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma}\right) dr_1}{\bar{e} z_h - (\beta\lambda_{h,\bar{e}}) \left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma(1+r)}\right) dr_1}$$

Taking the derivative with respect to dr_1, we obtain

$$\frac{d\frac{\mathbb{E}[c_{h,1}]}{c_{h,0}}}{dr_1} = \frac{\Big(\Pi_{\bar{e}\bar{e}} m_{\bar{e}} + (1 - \Pi_{\bar{e}\bar{e}})\Big) (\beta\lambda_{h,\bar{e}}) \left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma}\right) c_{h,0} + \mathbb{E}[c_{h,1}] (\beta\lambda_{h,\bar{e}}) \left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma(1+r)}\right)}{(\bar{e} z_h - (\beta\lambda_{h,\bar{e}}) \left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma(1+r)}\right) dr_1)^2}.$$

Which simplifies to

$$\frac{d\frac{\mathbb{E}[c_{h,1}]}{c_{h,0}}}{dr_1} = (\beta\lambda_{h,\bar{e}}) \left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma}\right) \frac{\Big(\Pi_{\bar{e}\bar{e}} m_{\bar{e}} + (1 - \Pi_{\bar{e}\bar{e}})\Big) c_{h,0} + \mathbb{E}[c_{h,1}]/(1 + r)}{(\bar{e} z_h - (\beta\lambda_{h,\bar{e}}) \left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma(1+r)}\right) dr_1)^2}$$

Now, taking the limit as $dr_1 \to 0$, we get

$$\frac{d\frac{\mathbb{E}[c_{h,1}]}{c_{h,0}}}{dr_1} = (\beta\lambda_{h,\bar{e}}) \left(z_h \frac{\bar{e}\rho(\bar{e})}{\sigma(1 + r)}\right) \frac{\Big(\Pi_{\bar{e}\bar{e}} m_{\bar{e}} + (1 - \Pi_{\bar{e}\bar{e}})\Big) c_{h,0} + \mathbb{E}[c_{h,1}]/(1 + r)}{(\bar{e} z_h)^2}$$

Which simplifies to

$$\frac{d\frac{\mathbb{E}[c_{h,1}]}{c_{h,0}}}{dr_1} = (\beta\lambda_{h,\bar{e}}) \left(\bar{e} \frac{\rho(\bar{e})}{\sigma}\right) \frac{\Big(\Pi_{\bar{e}\bar{e}} m_{\bar{e}} + (1 - \Pi_{\bar{e}\bar{e}})\Big) c_{h,0} + \mathbb{E}[c_{h,1}]/(1 + r)}{\bar{e} z_h}$$

Computing the EIS:

$$\text{EIS} = \frac{1 + r}{\frac{\mathbb{E}[c_{h,1}]}{c_{h,0}}} \frac{d\frac{\mathbb{E}[c_{h,1}]}{c_{h,0}}}{dr_1} = (1 + r)\beta\lambda_{h,\bar{e}} \left(\frac{\bar{e}\rho(\bar{e})}{\sigma}\right) \frac{\Big(\Pi_{\bar{e}\bar{e}} m_{\bar{e}} + (1 - \Pi_{\bar{e}\bar{e}})\Big) c_{h,0} + \mathbb{E}[c_{h,1}]/(1 + r)}{\bar{e} z_h \mathbb{E}[e'|\bar{e}]}.$$

Plugging the values for consumptions and given that $m_{\bar{e}} = 1 - \frac{\lambda}{\Pi_{\bar{e}\bar{e}}(1+r)}$,

$$\text{EIS} = \rho(\bar{e})(1+r)\beta\frac{\lambda}{\Pi_{\bar{e}\bar{e}}}\left[\left(1-\frac{\lambda}{1+r}\right)\frac{\bar{e}}{\mathbb{E}[e'|\bar{e}]}+\frac{1}{1+r}\right]\frac{1}{\sigma}.$$

Notice that, in the absence of taste for wealth and idiosyncratic shocks, the EIS collapses to $1/\sigma$.

The sign of the derivative is given by:

$$\frac{d\lambda}{dz_h}\left\{\left[\left(1-\frac{\lambda}{1+r}\right)\bar{e}+\mathbb{E}[e'|\bar{e}]/(1+r)\right]-\frac{\lambda}{1+r}\bar{e}\right\} = \underbrace{\frac{d\lambda}{dz_h}}_{<0}\underbrace{\left[\left(1-\frac{\lambda}{1+r}\right)\bar{e}+\frac{\mathbb{E}[e'|\bar{e}]-\lambda}{1+r}\right]}_{>0} < 0.$$

The elasticity of intertemporal substitution is a decreasing function of permanent labor income.

C.3 Indirect effect

The indirect effect of a labor demand shock is given by :

$$\text{Indirect effect} = \mathbf{M}^n d\mathbf{N}.$$

Taking the first element of the vector:

$$\text{Indirect effect}_0 = dN_0 - (1-\mu)\frac{\omega_h z_h}{1+r}\frac{\lambda(1-\rho)}{1-\beta\lambda\rho}dN_0.$$

Taking the derivative of the indirect effect with respect to z_h:

$$\begin{aligned}\frac{d\text{Indirect effect}_0}{dz_h} =& \underbrace{-(1-\mu)\frac{\omega_h}{1+r}\frac{dN_0}{1-\beta\lambda\rho}\lambda(1-\rho)}_{\text{composition effect}<0}\\ &-(1-\mu)\frac{\omega_h z_h}{1+r}(1-\rho)dN_0\left(\frac{\frac{d\lambda}{dz_h}(1-\beta\lambda\rho)+\beta\rho\frac{d\lambda}{dz_h}\lambda}{(1-\beta\lambda\rho)^2}-\frac{dr}{dz_h}\frac{1}{1+r}\frac{1}{1-\beta\lambda\rho}\lambda\right)\end{aligned}$$

$$\begin{aligned}\frac{d\text{Indirect effect}_0}{dz_h} =& \underbrace{-(1-\mu)\frac{\omega_h}{1+r}\frac{dN_0}{1-\beta\lambda\rho}\lambda(1-\rho)}_{\text{composition effect }<0}\\ &\underbrace{-(1-\mu)\frac{\omega_h z_h}{1+r}\frac{1-\rho}{1-\beta\lambda\rho}dN_0\left(\frac{d\lambda}{dz_h}\frac{1}{1-\beta\lambda\rho}-\frac{dr}{dz_h}\frac{\lambda}{1+r}\right)}_{\text{behavior effect }>0}\end{aligned}$$

D Matrices Summary

D.1 Sequence-space Jacobians in zero-liquidity HANK

D.1.1 With homothetic preferences and no idiosyncratic risk

The Euler equation holds with equality for all households. The aggregate sequence-space Jacobians are

$$\mathbf{A}^{\tilde{n}} = \beta \begin{pmatrix} 1 & -(1-\beta) & -(1-\beta)\beta & -(1-\beta)\beta^2 & \cdots \\ 1 & \beta & -\left(1-\beta^2\right) & -\left(1-\beta^2\right)\beta & \cdots \\ 1 & \beta & \beta^2 & -\left(1-\beta^3\right) & \ddots \\ \vdots & \vdots & \vdots & \beta^3 & \ddots \end{pmatrix} \text{ and } \mathbf{A}^r = \frac{\beta}{1+r}\frac{1}{\sigma}\begin{pmatrix} 0 & 1 & \beta & \cdots \\ 0 & 1 & 1+\beta & \cdots \\ 0 & 1 & 1+\beta & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

With element t, s of $\mathbf{A}^{\tilde{n}}$ being $\frac{\partial a_{t+1}}{\partial(1-\tau_s)w_s}$ and element t, s of $\mathbf{A}^r$ being $\text{‘}\frac{\partial a_{t+1}}{\partial r_s}$.

$$\mathbf{M}^{\tilde{n}} = \begin{pmatrix} \frac{r}{1+r} & \frac{r}{(1+r)^2} & \frac{r}{(1+r)^3} & \cdots \\ \frac{r}{1+r} & \frac{r}{(1+r)^2} & \frac{r}{(1+r)^3} & \cdots \\ \frac{r}{1+r} & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \text{ and } \mathbf{M}^r = \frac{1}{(1+r)^2}\frac{1}{\sigma}\begin{pmatrix} 0 & -1 & -\frac{1}{1+r} & \cdots \\ 0 & r & -\frac{1}{1+r} & \cdots \\ 0 & r & \frac{r(2+r)}{1+r} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

With element t, s of $\mathbf{M}^{\tilde{n}}$ being $\frac{\partial c_t}{\partial(1-\tau_s)w_s}$ and element t, s of $\mathbf{M}^r$ being $\frac{\partial c_t}{\partial r_s}$.

D.1.2 With non-homothetic preferences and no idiosyncratic risk

The Euler equation holds with equality for all high-permanent type households. The aggregate sequence-space Jacobians are

$$\mathbf{A}^n = \frac{\omega_h z_h}{1+r}\begin{pmatrix} \lambda & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2 & \cdots \\ \lambda^2 & -\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda & -\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2-\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \cdots \\ \lambda^3 & -\lambda^2\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda^2 & -\lambda^2\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2-\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

$$\mathbf{M}^{\tilde{n}} = \frac{\omega_h z_h}{1+r}\begin{pmatrix} -\lambda & \left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2 & \cdots \\ (1+r)\lambda-\lambda^2 & (\lambda-(1+r))\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)-\lambda & \left(\beta\lambda^2-(1+r)\beta\lambda+1\right)\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \cdots \\ (1+r)\lambda^2-\lambda^3 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathbf{I}$$

$$\mathbf{A}^r = z_h\omega_h\frac{\beta\lambda}{1+r}\frac{1}{\sigma}\begin{pmatrix} 0 & 1 & \beta\lambda & \cdots \\ 0 & \lambda & 1+\beta\lambda^2 & \cdots \\ 0 & \lambda^2 & \lambda^2\beta\lambda+\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathbf{M}^r = z_h\omega_h\frac{\beta\lambda}{1+r}\frac{1}{\sigma}\begin{pmatrix} 0 & -1 & -\beta\lambda & \cdots \\ 0 & 1+r-\lambda & (1+r)\beta\lambda-1-\beta\lambda & \cdots \\ 0 & (1+r)\lambda-\lambda^2 & (1+r)(1+\beta\lambda^2)-\lambda^2\beta\lambda-\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

D.1.3 With homothetic preferences and idiosyncratic risk

The Euler equation holds with equality for all high idiosyncratic type households. The aggregate sequence-space Jacobians are

$$\mathbf{A}^n = \frac{1-\mu}{1+r}\begin{pmatrix} \lambda & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2 & \cdots \\ \lambda^2 & -\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda & -\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2-\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \cdots \\ \lambda^3 & -\lambda^2\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda^2 & -\lambda^2\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2-\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

$$\mathbf{M}^{\tilde{n}} = \frac{1-\mu}{1+r}\begin{pmatrix} -\lambda & \left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2 & \cdots \\ (1+r)\lambda-\lambda^2 & (\lambda-(1+r))\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)-\lambda & (\beta\lambda^2-(1+r)\beta\lambda+1)\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \cdots \\ (1+r)\lambda^2-\lambda^3 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}+\mathbf{I}$$

$$\mathbf{A}^r = (1-\mu)\frac{\beta\lambda}{\sigma(1+r)}\begin{pmatrix} 0 & 1 & \beta\lambda & \cdots \\ 0 & \lambda & 1+\beta\lambda^2 & \cdots \\ 0 & \lambda^2 & \lambda^2\beta\lambda+\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathbf{M}^r = (1-\mu)\frac{\beta\lambda}{\sigma(1+r)}\begin{pmatrix} 0 & -1 & -\beta\lambda & \cdots \\ 0 & 1+r-\lambda & (1+r)\beta\lambda-1-\beta\lambda & \cdots \\ 0 & (1+r)\lambda-\lambda^2 & (1+r)(1+\beta\lambda^2)-\lambda^2\beta\lambda-\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

D.1.4 With non-homothetic preferences and idiosyncratic risk

The Euler equation holds with equality for the high-idiosyncratic type households that have the highest permanent-income type. The aggregate sequence-space Jacobians are

$$\mathbf{A}^n = \omega_h z_h \frac{1-\mu}{1+r} \begin{pmatrix} \lambda & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & -\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2 & \cdots \\ \lambda^2 & -\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda & -\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2-\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \cdots \\ \lambda^3 & -\lambda^2\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda^2 & -\lambda^2\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2-\lambda\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)+\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

$$\mathbf{M}^{\tilde{n}} = \omega_h z_h \frac{1-\mu}{1+r} \begin{pmatrix} -\lambda & \left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)^2 & \cdots \\ (1+r)\lambda-\lambda^2 & (\lambda-(1+r))\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda)-\lambda & (\beta\lambda^2-(1+r)\beta\lambda+1)\left(\frac{1}{\beta}-\lambda\right)(\beta\lambda) & \cdots \\ (1+r)\lambda^2-\lambda^3 & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} + \mathbf{I}$$

$$\mathbf{A}^r = (1-\mu)\omega_h z_h \rho(\bar{e}) \frac{\beta\lambda}{\sigma(1+r)} \begin{pmatrix} 0 & 1 & \beta\lambda & \cdots \\ 0 & \lambda & 1+\beta\lambda^2 & \cdots \\ 0 & \lambda^2 & \lambda^2\beta\lambda+\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$$\mathbf{M}^r = (1-\mu)\omega_h z_h \rho(\bar{e}) \frac{\beta\lambda}{\sigma(1+r)} \begin{pmatrix} 0 & -1 & -\beta\lambda & \cdots \\ 0 & 1+r-\lambda & (1+r)\beta\lambda-1-\beta\lambda & \cdots \\ 0 & (1+r)\lambda-\lambda^2 & (1+r)(1+\beta\lambda^2)-\lambda^2\beta\lambda-\lambda & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

D.2 Some useful matrices

$$\mathbf{F} = \begin{pmatrix} 0 & 1 & 0 & \cdots \\ 0 & 0 & 1 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & & & \ddots \end{pmatrix} \text{ and } \mathbf{L} = \begin{pmatrix} 0 & 0 & 0 & \cdots \\ 1 & 0 & 0 & \cdots \\ 0 & 1 & 0 & \cdots \\ \vdots & & & \ddots \end{pmatrix}$$

$$\mathbf{K} = -\sum_{t=1}^{\infty} \frac{\mathbf{F}^t}{(1+r)^t} = -\begin{pmatrix} 0 & 1/(1+r) & 0 & \cdots \\ 0 & 0 & 1/(1+r) & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & & & \ddots \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1/(1+r)^2 & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \vdots & & & \ddots \end{pmatrix} - ...$$

$$\mathbf{K} = -\begin{pmatrix} 0 & 1/(1+r) & 1/(1+r)^2 & 1/(1+r)^3 & \cdots \\ 0 & 0 & 1/(1+r) & 1/(1+r)^2 & \cdots \\ 0 & 0 & 0 & 1/(1+r) & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & & & & \ddots \end{pmatrix}$$

E Additional Material for One-asset Model

E.1 Decomposition of a monetary policy shock

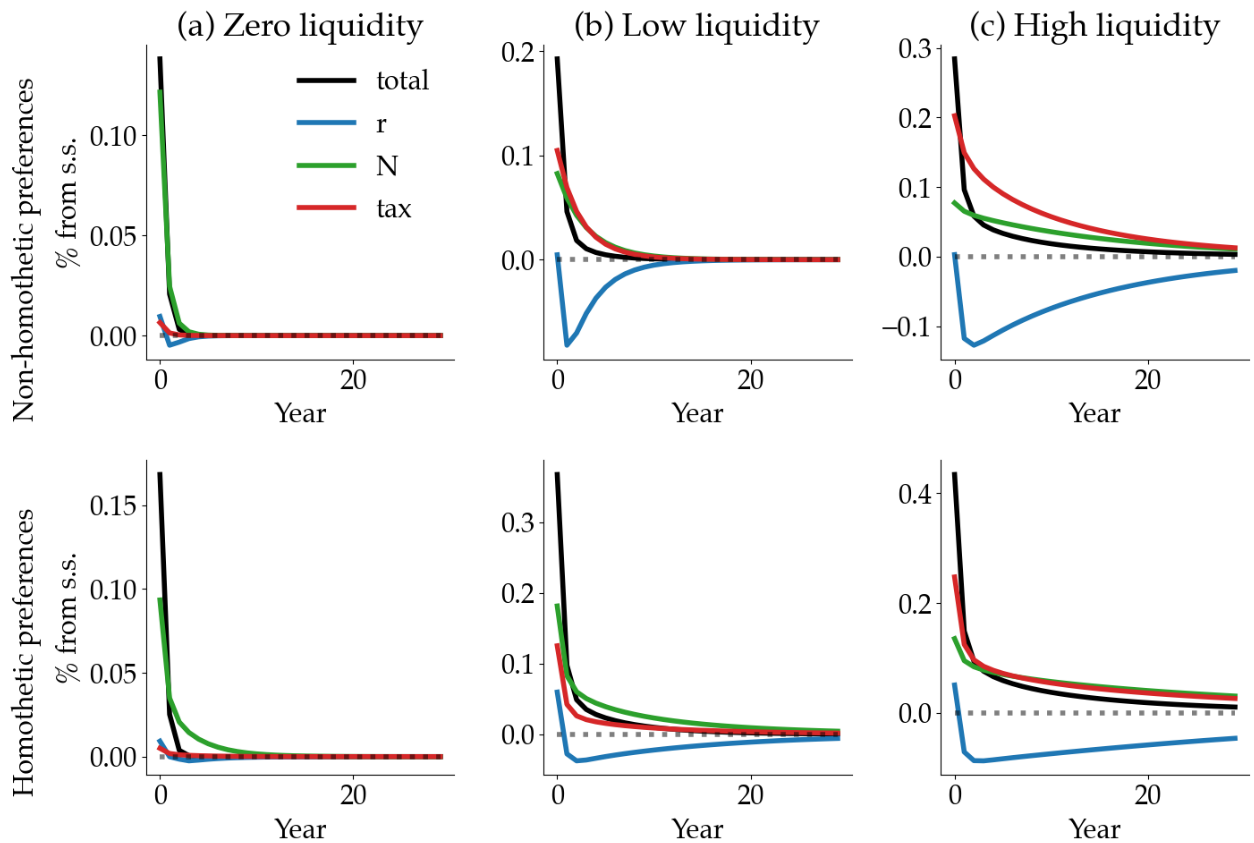

Figure 5: Decomposition of the effect of a monetary shock on output

Note: This figure plots the aggregate effect and the transmission channels of a monetary policy shock for different levels of government bonds (from zero on the left, to a low level in the middle, and to high a level on the right). On top, the aggregate effect and the transmission channels are computed with non-homothetic preferences for wealth while at the bottom they are computed without non-homothetic preferences for wealth.

E.2 Rise in permanent labor income inequality with homothetic preferences

We set $\gamma = 0$ and study the effect of permanent labor income inequality on the transmission of a monetary policy shock. We find that an increase in permanent labor income has almost no effect on the output response to a monetary policy shock, even in the presence of a positive supply of liquidity. Indeed, as shown by Straub (2019) in Lemma 1, steady-state policy functions in a model with homothetic preferences are linear in permanent income.

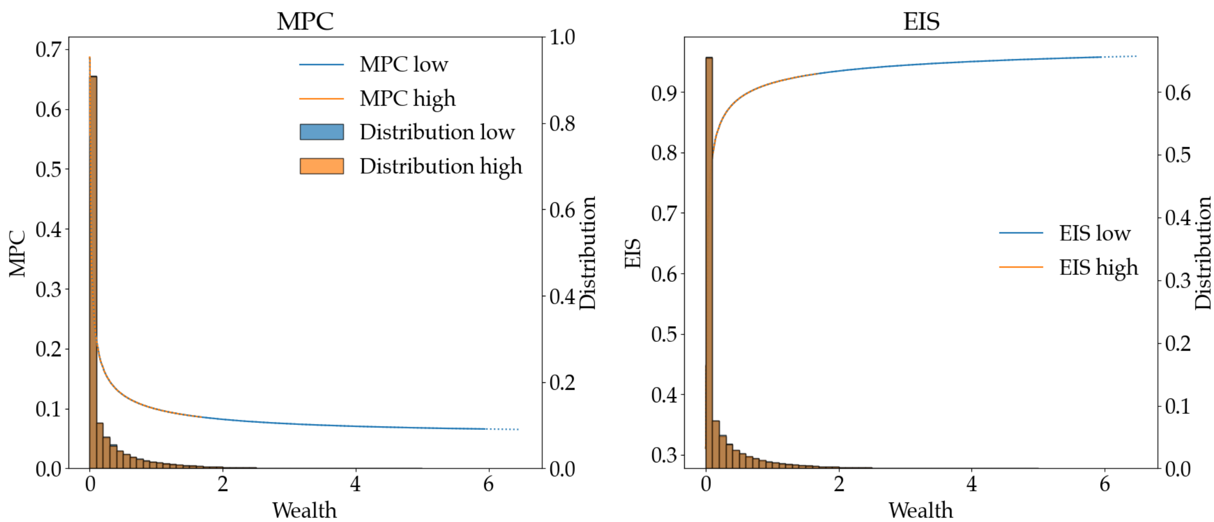

Figure 6: MPC and EIS along the wealth distribution

Note: This figure plots the marginal propensity to consume out of a one-time income shock (on the left) and the elasticity of intertemporal substitution (on the right) normalized by the level of permanent income for different levels of wealth (x-axis) and for different levels of permanent income (in orange, the permanent labor income of the top 10% in 1989, in dotted orange the permanent labor income of the top 10% in 2019; in blue, the permanent labor income of the bottom 90% in 1989, in dotted blue the permanent labor income of the bottom 90% in 2019. We also plot the wealth distributions of the two permanent income types normalized (in blue, the low-income type, and in orange, the high-income type) before and after the rise in permanent labor income inequality (in dark color before and transparent after the rise). B is fixed at the low-liquidity value (0.23). Notice that the two normalized distributions are perfectly equal to each other.

Figure 6 gives us a visual representation of this neutrality result. In the presence of homothetic preferences, the MPC and the EIS are still functions of permanent income. But they are now *linear* functions of permanent income. The EIS and the MPC normalized by permanent income are hence equal across permanent income types. Redistributing permanent income across households leaves the aggregate MPC and the aggregate EIS constant. Similarly, wealth distributions normalized by the level of permanent income are equal across types.

This neutrality result on both the aggregate effect and the transmission channels of the permanent labor income distribution is confirmed in Figure 7.

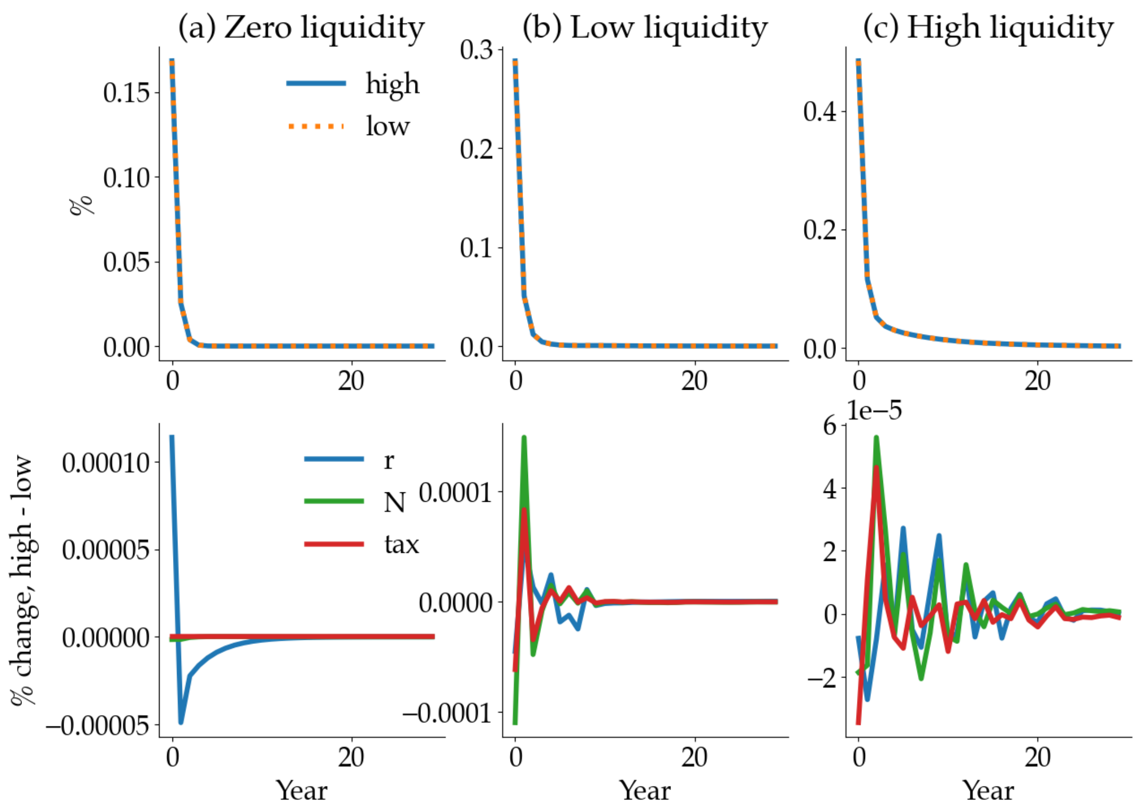

Figure 7: Decomposition of the effect of a monetary shock on output

E.3 Rise in the variance of idiosyncratic shocks

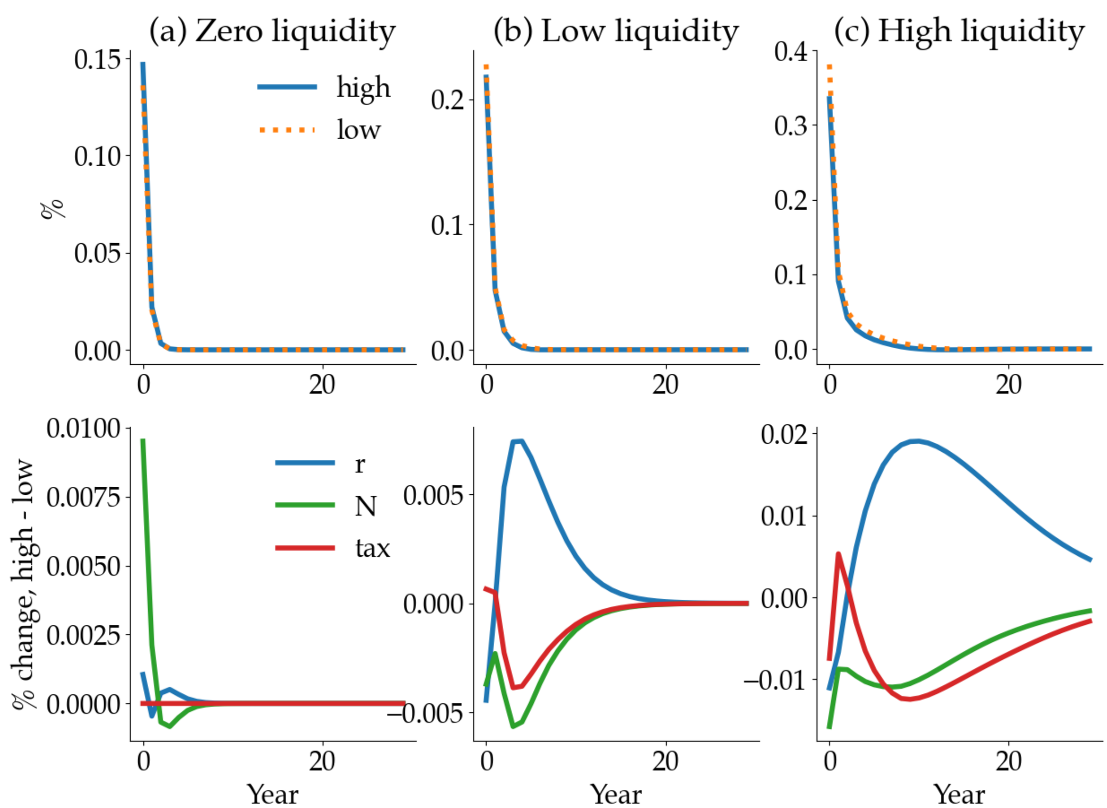

Figure 8: Decomposition of the effect of a monetary shock on output

F Additional Material for Two-asset Model

F.1 Intermediate firm's problem in two-asset HANK

Intermediate firms choose prices, labor, and capital next period so as to maximize their

$$J_t(k_t) = \max_{\mathcal{P}_t, k_{t+1}, n_t} \left\{ \frac{\mathcal{P}_t}{P_t} F(k_t, n_t) - \frac{W_t}{P_t} n_t - i_t - \varphi\left(\frac{k_{t+1}}{k_t}\right) k_t - \xi\left(\mathcal{P}_t, \mathcal{P}_{t-1}\right) Y_t + \frac{1}{1 + r_{t+1}} J_{t+1}(k_{t+1}) \right\},$$

$$\text{with investment } i_t = k_{t+1} - (1-\delta) k_t,$$

$$\text{subject to the final-goods firm's demand: } F(k_t, n_t) = \left(\frac{\mathcal{P}_t}{P_t}\right)^{-\mu^p/(\mu^p - 1)} Y_t.$$

$$\text{Given the Rotemberg adjustment costs: } \xi\left(\mathcal{P}_t, \mathcal{P}_{t-1}\right) \equiv \frac{1}{2\kappa^p(\mu^p - 1)} \left(\frac{\mathcal{P}_t - \mathcal{P}_{t-1}}{\mathcal{P}_{t-1}}\right)^2,$$

Year	bottom50	next40	top10	top100
1989	20%	51%	28%	8%
2019	18%	48%	34%	11%

Table 8: Distribution of permanent income used to calibrate the model

Year	Bottom 50%	Next 40%	Next 9%	Top 1%
1989	3%	28%	37%	33%
2019	2%	20%	38%	40%

Table 9: Distribution of illiquid wealth in the Survey of Consumer Finance used to calibrate the model

And the quadratic capital adjustment costs: $\varphi\left(\frac{k_{t+1}}{k_t}\right)k_t$ with $\varphi(x) \equiv \frac{1}{2\delta\varepsilon_I}(x-1)^2$.

With ε_I, the sensitivity of gross investment to the Tobin's Q.

F.2 Distribution of permanent income and illiquid wealth

Tables 8 and 9 describe respectively the distribution of labor income from Piketty et al. (2018) and the distribution of illiquid wealth in the Survey of Consumer finance that we use to calibrate our parameters $(s_{b50}, s_{n40}, s_{n9}, s_{top1})$ and $(a_{b50}, a_{n40}, a_{n9}, a_{top1})$. To move from the labor shares to the parameters s, we divide them by the weight of each household type. For the illiquid wealth, we divide them by the share of illiquid wealth and multiply them by the total amount of illiquid wealth in the economy.

F.3 Two-asset with constant portfolio

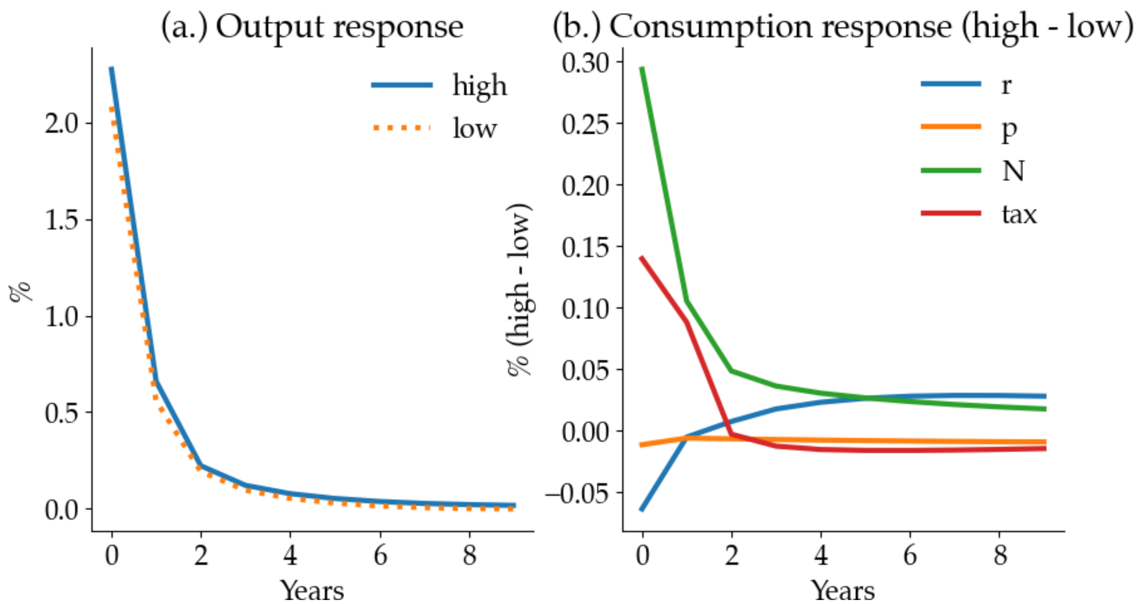

Figure 9: Output reponse and Decomposition

Note: The left figure plots the output response following a monetary shock. The right figure plots the difference in the decomposition between the high-inequality economy minus the low-inequality economy.

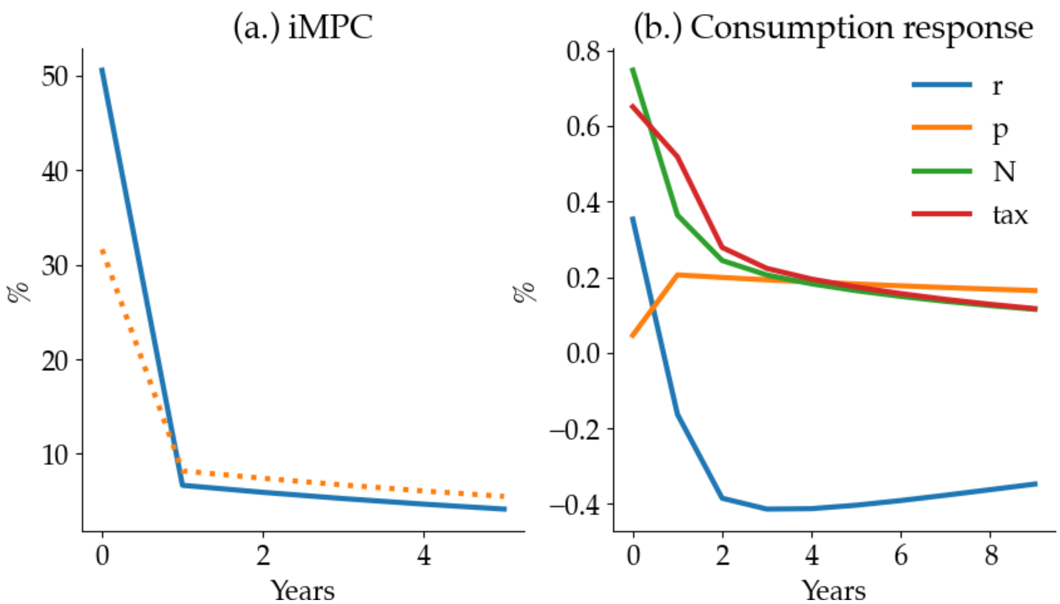

Figure 10: iMPC and decomposition

Note: The left figure plots the iMPC. The right figure plots the decomposition of a monetary policy shock in the high-inequality economy.

F.4 Computational details

We solve the HANK model of Section 2 and the quantitative model of Section 3 using the Sequence-Space Jacobian method of Auclert et al. (2021), and their package available online.

To solve for the steady-state, we fix the interest rate at $r = 5\%$ and find a β to clear the asset with a bisection method, using the endogenous-grid method to solve the problem of the household.

In the one-asset HANK model, we solve the problem of the household on a grid of 500 assets points and discretize the AR(1) process for idiosyncratic shock using the Rouwenhorst method with 11 points.

In the two-assets HANK model, we solve the problem of the household on a grid of 50 points for the illiquid assets and 50 points for the illiquid assets. The productivity process is discretized on a grid of 5 points.